

Combining Partial Sums and FHT for the Fastest Known Attack on 6-Round AES

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Plan of this Section

1. Motivation

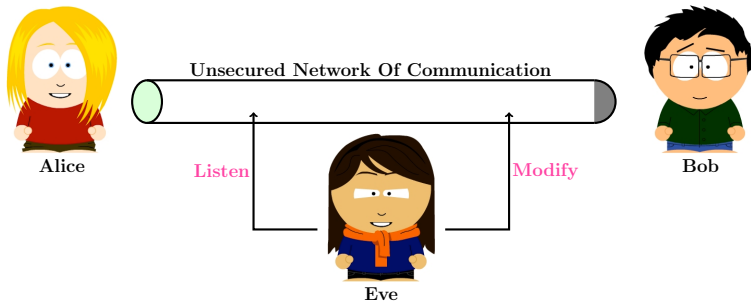
Why Symmetric Key Cryptanalysis?

Algorithm For Key Recovery

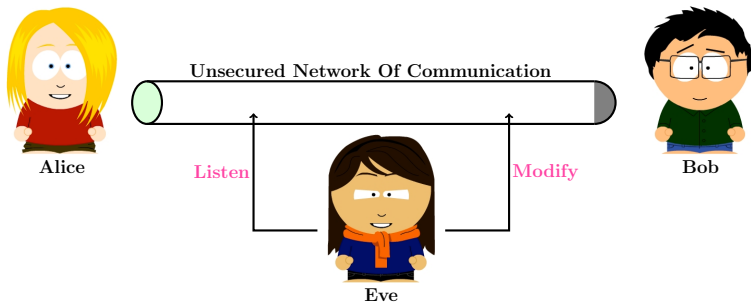
2. Integral Attack On AES

3. Partial Sums Meet FHT (Eurocrypt 2024)

Why Cryptography?



Why Cryptography?



Goal: Efficient algorithms with the following functionalities:

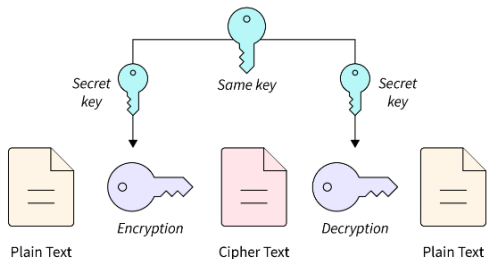
Confidentiality

Integrity

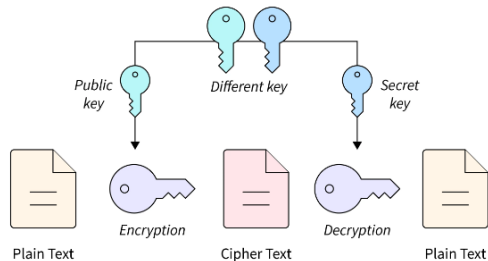
Authenticity

Symmetric and Asymmetric Key Cryptography

Symmetric Encryption



Asymmetric Encryption



Symmetric-Key Cryptography

Symmetric-Key Cryptosystem

$\mathcal{P}$

$\mathcal{K}$

$\mathcal{C}$

$\mathcal{E}$

$\mathcal{D}$

$$\forall P \in \mathcal{P}, K \in \mathcal{K}, \\ \mathcal{D}_K(\mathcal{E}_K(P)) = P$$

Symmetric-Key Cryptography

Symmetric-Key Cryptosystem

$\mathcal{P}$

$\mathcal{K}$

$\mathcal{C}$

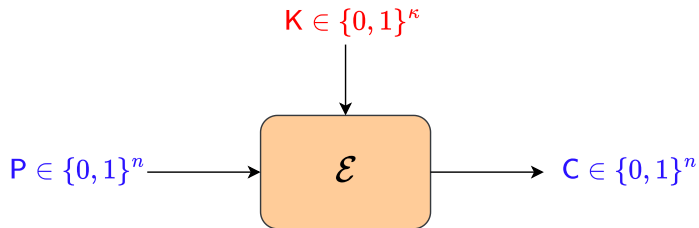
$\mathcal{E}$

$\mathcal{D}$

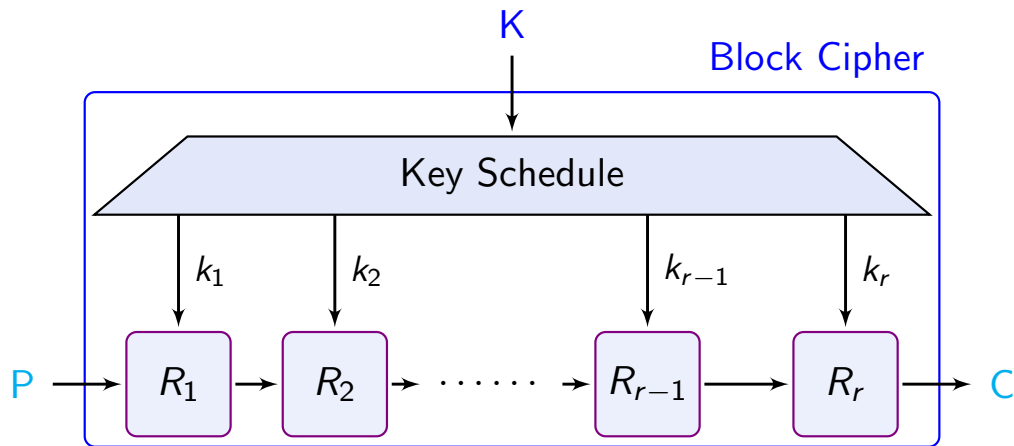
$$\forall P \in \mathcal{P}, K \in \mathcal{K}, \\ \mathcal{D}_K(\mathcal{E}_K(P)) = P$$

e.g., Block ciphers, stream ciphers, hash functions, MACs, ...

Block Cipher

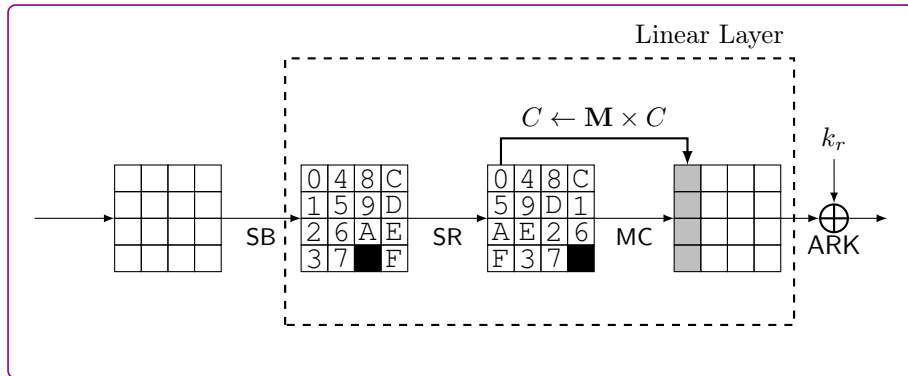


Iterated Block Cipher



AES: The NIST Standard

Round function

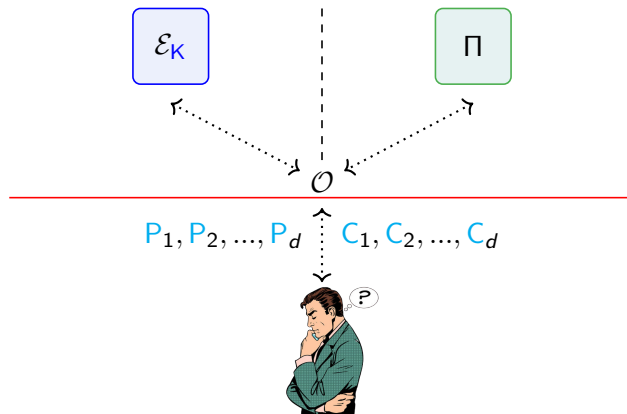


Motivation

Why Symmetric Key Cryptanalysis?

Cryptanalysis Perspective: Left-Right Distinguishing Game

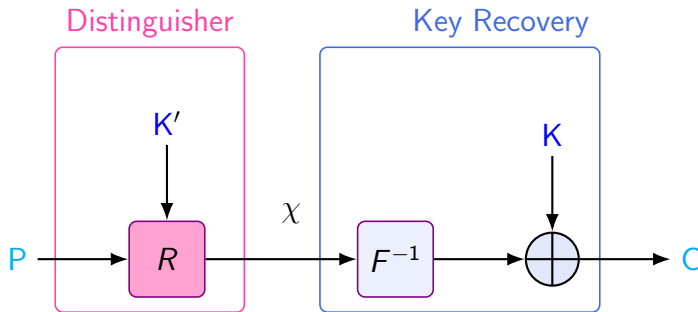
$\mathcal{E}: \mathcal{P} \times \mathcal{K} \rightarrow \mathcal{C}$ is a block cipher and Π is a random permutation

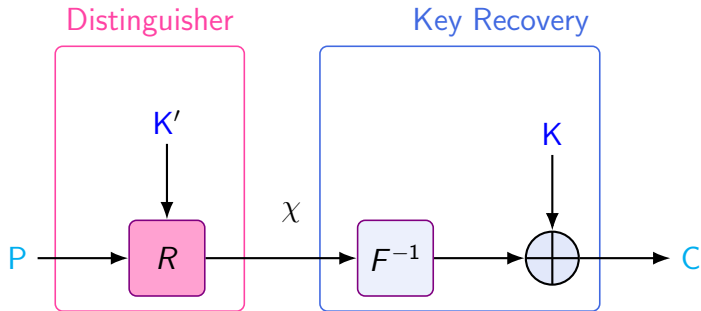


“Searching ‘Distinguisher’” | Key Recovery

My Contributions in Cryptanalysis

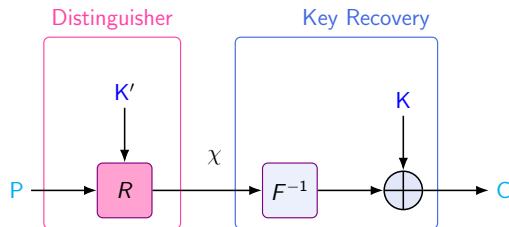
Primitive	Distinguisher	Key Recovery	Venue
AES	Integral	✓	Eurocrypt 24
QARMA	Differential, Weak-Key	✓	FSE 25
ARADI	Integral, Differential	✓	Under Submission
PRESENT, GIFT, BAKSHEESH	Impossible Differential	✓	CHES 25
PRESENT, GIFT	Integral	✓	ACNS 23
TinyJambu-192/256	Differential	✓	FSE 23
TinyJambu-128	Integral		Indocrypt 22
KNOT, ASCON	Integral		Latincrypt 21
SHA-3, Xoodyak, Bash	Integral		DCC 25
QARMAv2	Known Attacks	✓	FSE 23
ChiLow	Known Attacks	✓	Eurocrypt 25
FRAST	Differential, Weak-key	✓	Under Submission





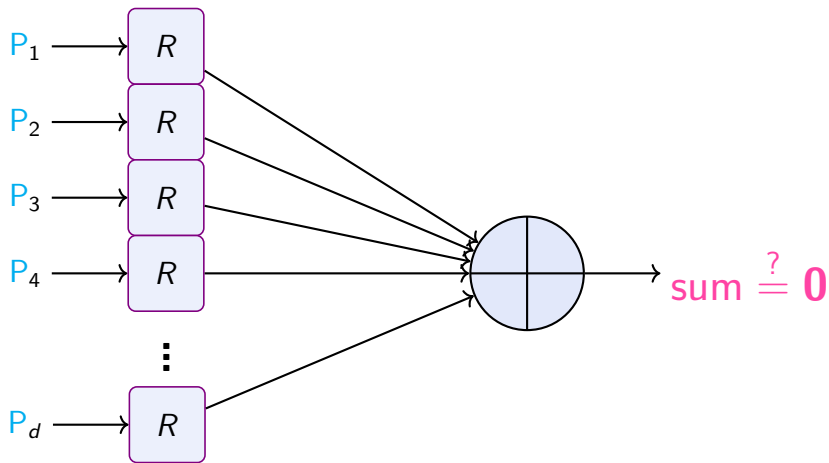
- $R(\cdot) = R_{r-1} \circ R_{r-2} \circ \dots \circ R_1 \circ R_0(\cdot)$

Classic Distinguishers in Cryptanalysis

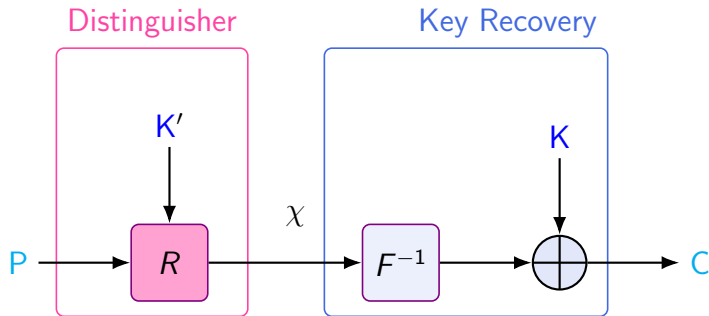


- **Differential** : $\Pr \left(R(P) \oplus R(P \oplus \Delta_{in}) = \Delta_{out} \right) = 2^{-a}, a \ll n$
- **Integral** : $\Pr \left(\bigoplus_{P \in \mathcal{I}} R(P) = \mathcal{S} \right) = 2^{-a}, a \ll n$
- **Linear** : $\left| \Pr \left(\alpha \cdot P \oplus \beta \cdot R(P) = 0 \right) - \frac{1}{2} \right| = 2^{-a}, a \ll n$

Integral/Zero-Sum Distinguisher



Key Recovery Using Integral/Zero-Sum Distinguisher



$$\bigoplus_{X \in \mathcal{X}} X = \bigoplus_{P \in \mathcal{P}} R(P) = 0 = \bigoplus_{C \in \mathcal{C}} F(C \oplus K), \text{ for the correct key } K$$

Motivation

Algorithm For Key Recovery

Key Recovery Algorithm

```
procedure FOO( $\mathcal{C} \subseteq \{0, 1\}^m$  of size  $2^m$ )  
  for  $K \in \{0, 1\}^m$  do  
     $S = 0$   
    for  $C \in \mathcal{C}$  do  
       $S = S \oplus F(C \oplus K)$   
    if  $S \neq 0$  then  
      Discard  $K$ 
```

Key Recovery Algorithm

procedure FOO($\mathcal{C} \subseteq \{0, 1\}^m$ of size 2^m)

for $K \in \{0, 1\}^m$ **do**

$S = 0$

for $C \in \mathcal{C}$ **do**

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if $S \neq 0$ **then**

 Discard K

procedure BAR($\mathcal{C} \subseteq \{0, 1\}^m$ of size 2^m)

for $K \in \{0, 1\}^m$ **do**

$S = 0$

for $C \in \{0, 1\}^m$ **do**

$S = S \oplus F(C \oplus K)G_{\mathcal{C}}(C)$

if $S \neq 0$ **then**

 Discard K

$$G_{\mathcal{C}}(C) = \begin{cases} 1, & \text{if occurrences of } C \text{ is odd in } \mathcal{C} \\ 0, & \text{if occurrences of } C \text{ is even in } \mathcal{C} \end{cases}$$

Key Recovery Algorithm

procedure FOO($\mathcal{C} \subseteq \{0, 1\}^m$ of size 2^m)

for $K \in \{0, 1\}^m$ **do**

$S = 0$

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procedure BAR($\mathcal{C} \subseteq \{0, 1\}^m$ of size 2^m)

for $K \in \{0, 1\}^m$ **do**

$S = 0$

for $C \in \{0, 1\}^m$ **do**

$S = S \oplus F(C \oplus K)G_{\mathcal{C}}(C)$

if $S \neq 0$ **then**

 Discard K

BAR $\equiv$ Convolution for each K ,

$$F * G_{\mathcal{C}}(K) = \bigoplus_{C \in \{0, 1\}^m} F(C \oplus K)G_{\mathcal{C}}(C)$$

The BAR matrix

$$F * G_C(\mathbf{K}) = \bigoplus_{\mathbf{C} \in \{0,1\}^m} F(\mathbf{K} \oplus \mathbf{C}) G_C(\mathbf{C})$$

$$\begin{bmatrix} F(0 \oplus 0) & F(0 \oplus 1) & F(0 \oplus 2) & F(0 \oplus 3) & F(0 \oplus 4) & F(0 \oplus 5) & F(0 \oplus 6) & F(0 \oplus 7) \\ F(1 \oplus 0) & F(1 \oplus 1) & F(1 \oplus 2) & F(1 \oplus 3) & F(1 \oplus 4) & F(1 \oplus 5) & F(1 \oplus 6) & F(1 \oplus 7) \\ F(2 \oplus 0) & F(2 \oplus 1) & F(2 \oplus 2) & F(2 \oplus 3) & F(2 \oplus 4) & F(2 \oplus 5) & F(2 \oplus 6) & F(2 \oplus 7) \\ F(3 \oplus 0) & F(3 \oplus 1) & F(3 \oplus 2) & F(3 \oplus 3) & F(3 \oplus 4) & F(3 \oplus 5) & F(3 \oplus 6) & F(3 \oplus 7) \\ F(4 \oplus 0) & F(4 \oplus 1) & F(4 \oplus 2) & F(4 \oplus 3) & F(4 \oplus 4) & F(4 \oplus 5) & F(4 \oplus 6) & F(4 \oplus 7) \\ F(5 \oplus 0) & F(5 \oplus 1) & F(5 \oplus 2) & F(5 \oplus 3) & F(5 \oplus 4) & F(5 \oplus 5) & F(5 \oplus 6) & F(5 \oplus 7) \\ F(6 \oplus 0) & F(6 \oplus 1) & F(6 \oplus 2) & F(6 \oplus 3) & F(6 \oplus 4) & F(6 \oplus 5) & F(6 \oplus 6) & F(6 \oplus 7) \\ F(7 \oplus 0) & F(7 \oplus 1) & F(7 \oplus 2) & F(7 \oplus 3) & F(7 \oplus 4) & F(7 \oplus 5) & F(7 \oplus 6) & F(7 \oplus 7) \end{bmatrix} \times \begin{bmatrix} G_C(0) \\ G_C(1) \\ G_C(2) \\ G_C(3) \\ G_C(4) \\ G_C(5) \\ G_C(6) \\ G_C(7) \end{bmatrix}$$

The BAR matrix

$F(0 \oplus 0)$	$F(0 \oplus 1)$	$F(0 \oplus 2)$	$F(0 \oplus 3)$	$F(0 \oplus 4)$	$F(0 \oplus 5)$	$F(0 \oplus 6)$	$F(0 \oplus 7)$
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Properties of BAR matrix $\mathcal{M}_m$

- $\mathcal{M}_m = \frac{1}{2^m}(\mathcal{H}_m \times \mathcal{D} \times \mathcal{H}_m)$, $\mathcal{H}_m[i, j] = \frac{1}{2^{m/2}}(-1)^{i \cdot j}$

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$$\text{In General } \mathcal{H}_m = \frac{1}{2^{m/2}} \begin{bmatrix} \mathcal{H}_{m-1} & \mathcal{H}_{m-1} \\ \mathcal{H}_{m-1} & -\mathcal{H}_{m-1} \end{bmatrix}$$

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- **FHT**: A divide-and-conquer algorithm, complexity = $\mathcal{O}(m2^m)$

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$$\begin{aligned}\mathcal{M}_m \times \mathcal{C} &= \frac{1}{2^m}(\mathcal{H}_m \times \mathcal{D} \times \mathcal{H}_m) \times \mathcal{C} \\ &= \frac{1}{2^m}(\mathcal{H}_m \times ((\mathcal{H}_m \times \mathcal{M}_m^0) \star (\mathcal{H}_m \times \mathcal{C})))\end{aligned}$$

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- Complexity: $2^{2m} \rightarrow 4m2^m$

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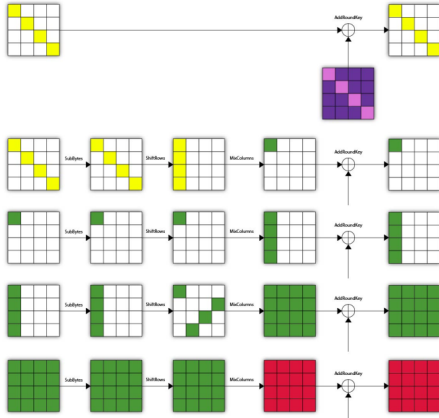
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- Complexity: $2^{2m} \rightarrow 4m2^m$
- For $m = 32$: $2^{64} \rightarrow 2^{39}$

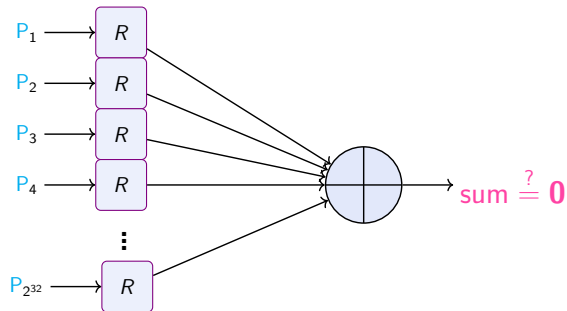
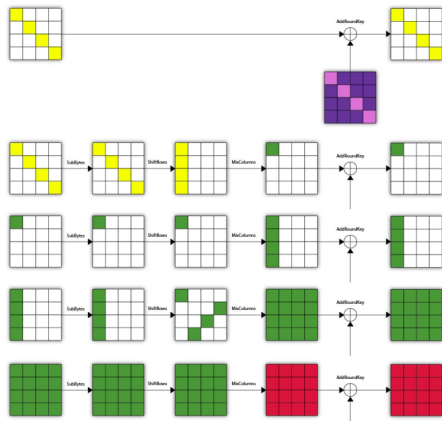
Plan of this Section

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 - FHT-based Integral Attack
 - Partial Sum Technique
3. Partial Sums Meet FHT (Eurocrypt 2024)

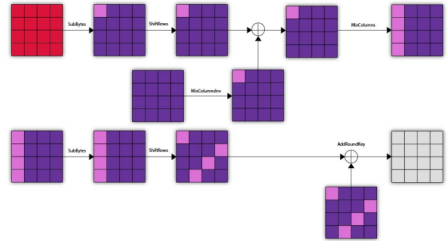
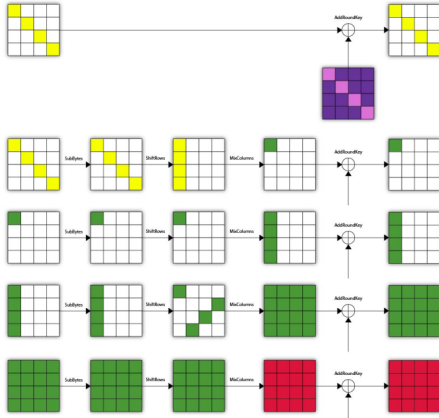
Zero-Sum Property On AES



Zero-Sum Property On AES

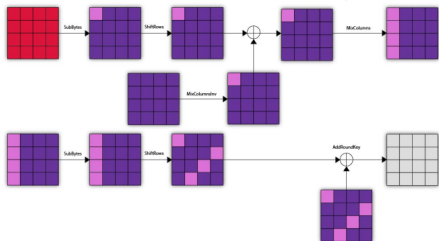


Zero-Sum Property On AES

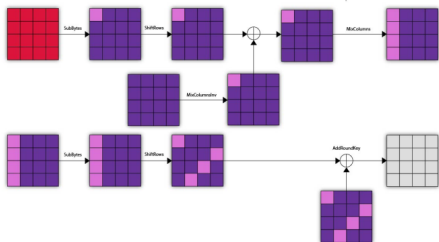


Integral Attack On AES

$$\chi(K, C) = S(K_4 \oplus S_3(K_3 \oplus C_3) \oplus S_2(K_2 \oplus C_2) \oplus S_1(K_1 \oplus C_1) \oplus S_0(K_0 \oplus C_0))$$



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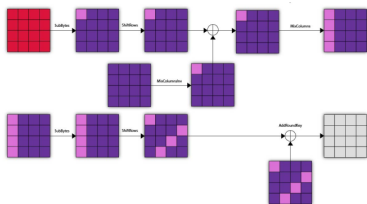
```

procedure FOO( $\mathcal{C} \subseteq \{0, 1\}^{32}$  of size  $2^{32}$ )
  for  $K = (K_4 K_3 K_2 K_1 K_0) \in \{0, 1\}^{40}$  do
     $S = 0$ 
    for  $C \in \mathcal{C}$  do
       $S = S \oplus \chi(K \oplus C)$ 
    if  $S \neq 0$  then Discard  $K$ 
  
```

Integral Attack On AES

FHT-based Integral Attack

Integral Attack On AES Using FHT [Todo et al. [TA14a]]

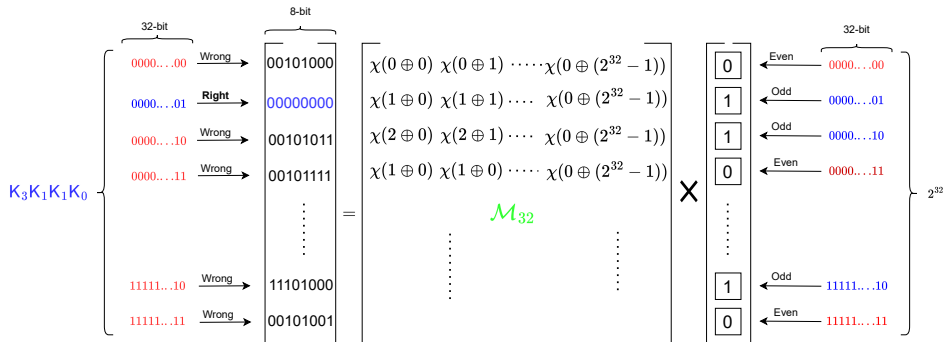


```

procedure BAR( $\mathcal{C} \subseteq \{0, 1\}^{32}$  of size  $2^{32}$ )
  for  $K_4 \in \{0, 1\}^8$  do
    for  $K = (K_3 K_2 K_1 K_0) \in \{0, 1\}^{32}$  do
       $S = 0$ 
      for  $C \in \{0, 1\}^{32}$  do
         $S = S \oplus \chi(K \oplus C) G_C(C)$ 
      if  $S \neq 0$  then
        Discard  $K$ 
  
```

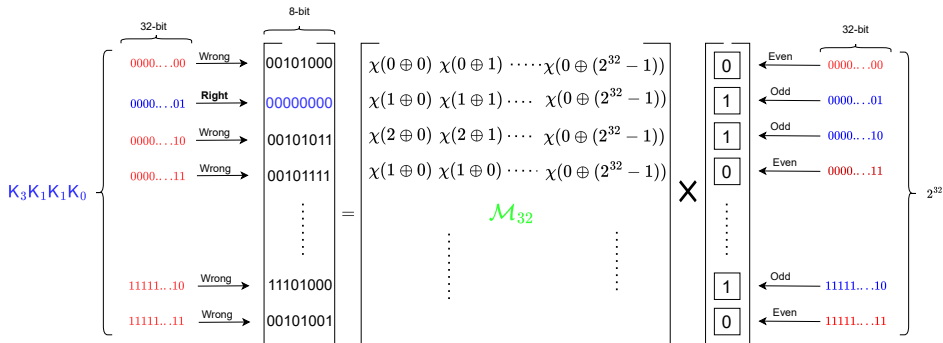
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For each $K_4 \in \{0, 1\}^8$:



Integral Attack On AES Using FHT [Todo et al. [TA14a]]

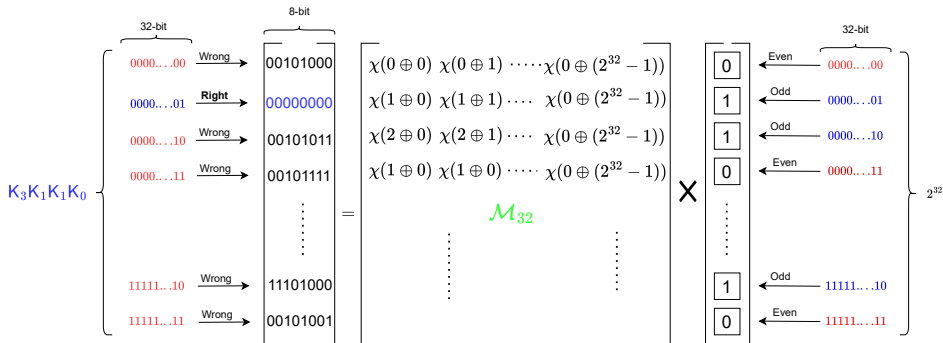
For each $K_4 \in \{0, 1\}^8$:



Time: $2^8 \times 2^{39}$

Integral Attack On AES Using FHT [Todo et al. [TA14a]]

For each $K_4 \in \{0, 1\}^8$:



Time: $2^8 \times 2^{39} \times 8$

The Problem with Finite Field Arithmetic

- $\chi * G(K) = \sum_C \chi(C \oplus K)G(C)$ VS $\chi * G(K) = \bigoplus_C \chi(C \oplus K)G(C)$
- We need functions whose output is an integer and not an element of $\mathbb{F}_2^8$

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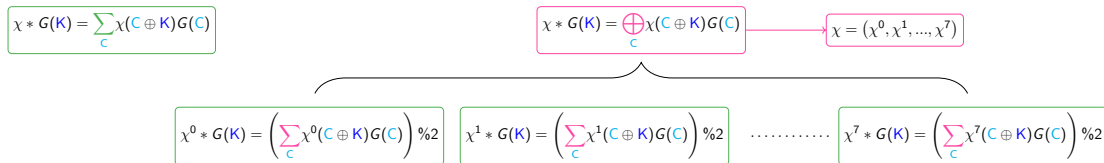
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$$\chi * G(K) = \sum_C \chi(C \oplus K)G(C)$$

$$\chi * G(K) = \bigoplus_C \chi(C \oplus K)G(C) \longrightarrow \chi = (\chi^0, \chi^1, \dots, \chi^7)$$

The Problem with Finite Field Arithmetic

- $\chi * G(\mathbf{K}) = \sum_{\mathbf{C}} \chi(\mathbf{C} \oplus \mathbf{K}) G(\mathbf{C})$ VS $\chi * G(\mathbf{K}) = \bigoplus_{\mathbf{C}} \chi(\mathbf{C} \oplus \mathbf{K}) G(\mathbf{C})$
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Integral Attack On AES

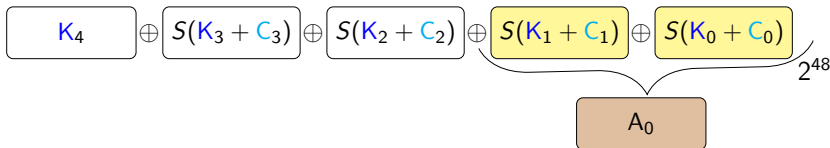
Partial Sum Technique

Partial Sum Technique [FKL⁺00]

$$\chi(K \oplus C) = S(K_4 \oplus S_3(K_3 \oplus C_3) \oplus S_2(K_2 \oplus C_2) \oplus S_1(K_1 \oplus C_1) \oplus S_0(K_0 \oplus C_0)))$$

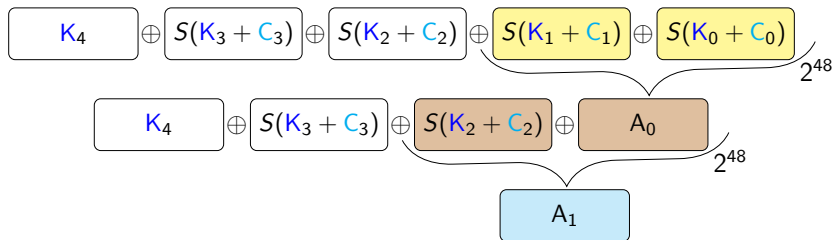
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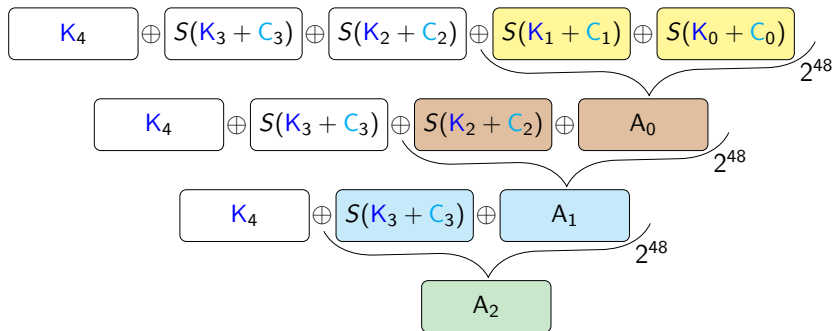
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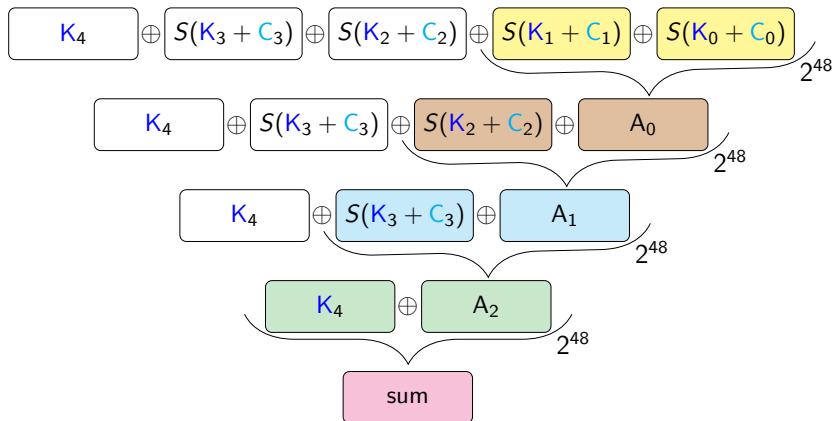
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Plan of this Section

1. Motivation
2. Integral Attack On AES
3. Partial Sums Meet FHT (Eurocrypt 2024)

Packing Technique

Results

Basic Idea

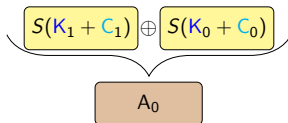
- We follow the general structure of the partial sums attack
- Replace each partial sum with FHT
- However, rearrange the steps to make it FHT compatible
- Rearrange the steps again to reduce memory complexity

Attacks on 6-Round AES

Cipher	Rounds	Data	Time	Technique and Source
AES	6	2^{32} CP	2^{71} Enc.	Square [DKR97]
		$6 \cdot 2^{32}$ CP	2^{52} S-box Eval.	Square & Partial sums [FKL ⁺ 01]
		2^{71} ACPC	2^{71} Enc.	Boomerang [Bir04]
		2^{33} CP	2^{52} S-box Eval.	Square & Partial sums [Tun12]
		$6 \cdot 2^{32}$ CP	2^{52} Add.	Square & FHT [TA14b]
		2^{26} CP	2^{80} Enc.	Mixture Differential [BDK ⁺ 20]
		2^{55} ACPC	2^{80} Enc.	Retracing Boomerang [DKRS20]
		$2^{79.7}$ ACPC	2^{78} Enc.	Boomeyong [RSP21]
		2^{59} ACPC	2^{61} Enc.	Truncated Boomerang [BL22]
		2^{33} CP	$2^{46.4}$ Add.	Square & Partial sums & FHT

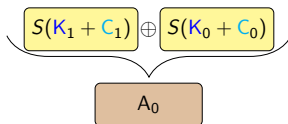
Partial Sums Meet FHT

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$A_0 = []$ of size $2^{16} \times 2^{24}$;

▷ 2^{40} memory

for all $(a_1, \mathbf{C}_2, \mathbf{C}_3) \in \{0, 1\}^{24}$ **do**

for all $(\mathbf{K}_0, \mathbf{K}_1) \in \{0, 1\}^{16}$ **do**

$$A_0[\mathbf{K}_0, \mathbf{K}_1][a_1, \mathbf{C}_2, \mathbf{C}_3] \leftarrow \bigoplus_{\mathbf{C}_0, \mathbf{C}_1} G_C[\mathbf{C}_0, \mathbf{C}_1, \mathbf{C}_2, \mathbf{C}_3] \cdot \mathbb{1}(S_0(\mathbf{C}_0 \oplus \mathbf{K}_0) \oplus S_1(\mathbf{C}_1 \oplus \mathbf{K}_1) = a_1)$$

Partial Sums Meet FHT

for all $(K_0, K_1) \in \{0, 1\}^{16}$ do

...

for all $K_2 \in \{0, 1\}^8$ do

...

for all $K_3 \in \{0, 1\}^8$ do

...

for all $K_4 \in \{0, 1\}^8$ do

$$\text{sum}[K_4] \leftarrow \bigoplus_{a_3} A_2[K_3][a_3] \cdot S(a_3 \oplus K_4)$$

for all $K_4 \in \{0, 1\}^8$ do

if $\text{sum}[K_4] \neq 0$ then

K_0, K_1, K_2, K_3, K_4 is not a valid key candidate

Complexities of Various Steps

Steps	Time (uint64_t Addition)	Memory (bits)
1	$2^{24} * (4 * 16 * 2^{16}) = 2^{46}$	2^{40}
2	$2^{16} * (2^8 * (4 * 16 * 2^{16})) = 2^{46}$	2^{24}
3	$2^{16} * 2^8 * (4 * 16 * 2^{16}) = 2^{46}$	2^{16}
4	$2^{16} * 2^8 * 2^8 * (8 * 4 * 8 * 2^8) = 2^{48}$	2^8
Total	$2^{48.8}$	2^{40}

Packing Multiple FHT's

for all $(K_0, K_1) \in \{0, 1\}^{16}$ do

...

for all $K_2 \in \{0, 1\}^8$ do

...

for all $K_3 \in \{0, 1\}^8$ do

...

for all $K_4 \in \{0, 1\}^8$ do

...

$$\text{sum}[k_4] \leftarrow \bigoplus_{a_3} S(a_3 \oplus K_4) \cdot A_2[K_3][a_3]$$

Partial Sums Meet FHT (Eurocrypt 2024)

Packing Technique

Packing Technique

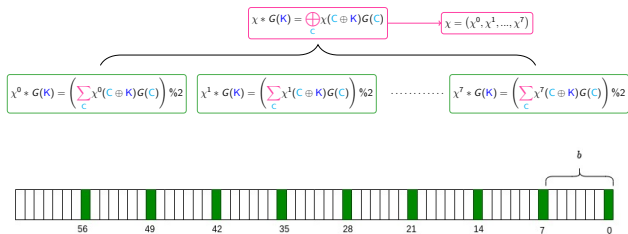
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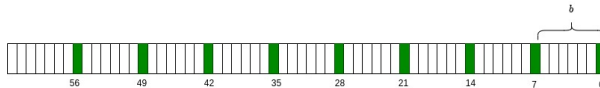
Packing Technique

- We assume that the attack is implemented using 64-bit operations in software
- Computing one convolution (results one bit information) is a waste of resources
- We compute several convolution in parallel and pack the results in 64-bit



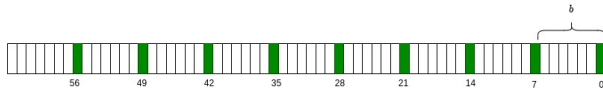
$$\bigoplus_{a_3} S(a_3 \oplus k_4) \cdot A_3[k_3][a_3] = \sum_{a_3} (2^{7b} S^7(K \oplus C) + \dots + S^0(K \oplus C)) A_3[k_3][a_3]$$

How large should b be?



How **Small** should b be?

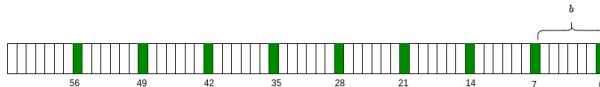
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- Ensure: $S^j(K \oplus C) < 2^b \forall j$?

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- $S^j(K \oplus C) \sim \text{Binomial}(128, 1/2)$
- $\mu = 64, \sigma = 4\sqrt{2}$

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How Large should b be?

- Ensure: No Overflow
- $7b \leq (64 - n) \implies b \leq 7$

Complexities of Various Steps

Steps	Time	Memory
1	$2^{46}/7$	2^{40}
2	$2^{46}/7$	2^{24}
3	$2^{46}/7$	2^{16}
4	$2^{48}/8$	2^8
Total	$\approx 2^{45.9}$	2^{40}

Complexity: Time $2^{45.9}$, Memory 128GB

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Complexity: Time $2^{45.9}$, Memory 128GB

Low Memory Variant: Time $2^{46.4}$, Memory 0.5GB

Partial Sums Meet FHT (Eurocrypt 2024)

Results

Integral Attack on 6-Round AES

	FHT+Part. Sums	FHT	Part. Sums
AWS Instance	m6i.32xlarge	r6i.32xlarge	m6i.32xlarge
Running Time(m)	48	3120	4859
Total Cost (USD)	5	418	497

In Conclusion: Our attack is **65** times faster and **83** times cheaper

The Improvement Matrix

	AES	Kuznyechik		MISTY1	CLEFIA
Rounds	6	6	7	8 (Full)	12
Improvement Factor	2^5	2^6	2^6	2^3	2^{30}

Conclusion

Partial Sum Meet FHT Without Packing VS Todo et al's FHT	Partial Sum Meet FHT Without Packing VS Partial Sum
6 or 12 times Improvement	8 times Improvement (One Sbox $\approx$ One 32-bit Addition)
20 times Improvement	60 times Improvement (16 parallel Sbox using AESNI)
Partial Sum Meet FHT With Packing VS Todo et al's FHT With Packing	Partial Sum Meet FHT With Packing VS Partial Sum With Parallel Sbox

- FFT-based techniques for ciphers over prime fields ($GF(p)$)

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- Can similar packing techniques be applied in other contexts?



https://github.com/ShibamCrS/Partial_Sums_Meet_FFT.git



<https://artifacts.iacr.org/eurocrypt/2024/a7/>

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