Finding Complete Impossible Differential Attacks on AndRX and ARX Designs

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Motivations

Research gap

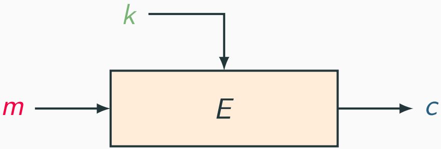
- 😰 Lack of automatic tool to find full Impossible Differential for AndRX and ARX ciphers

Our contributions

- ☺ Expanded [Hadipour et al., 2024]'s method
  - Handling indirect contradictions
  - Adaptation for AndRX and ARX designs
- ☺ Proposed a unified model to combine both distinguisher identification and key recovery for AndRX designs

Block Ciphers

- Encrypt plaintext m into cipher text c using key k.
- Generates a family of 2^k permutations indexed by the key k.

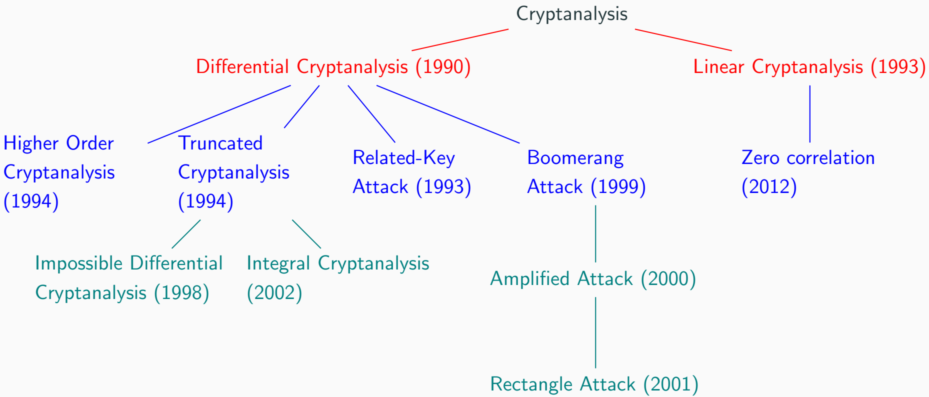


$$E : \{0,1\}^n \times \{0,1\}^k \to \{0,1\}^n$$

Cryptanalysis of block ciphers

- Exhaustive search: try all 2^k possible keys.
- Secure if no attack faster.
- Various other attacks.

Variants of Cryptanalysis



Cryptanalysis

Differential Cryptanalysis (1990)

Linear Cryptanalysis (1993)

Higher Order Cryptanalysis (1994)

Truncated Cryptanalysis (1994)

Related-Key Attack (1993)

Boomerang Attack (1999)

Zero correlation (2012)

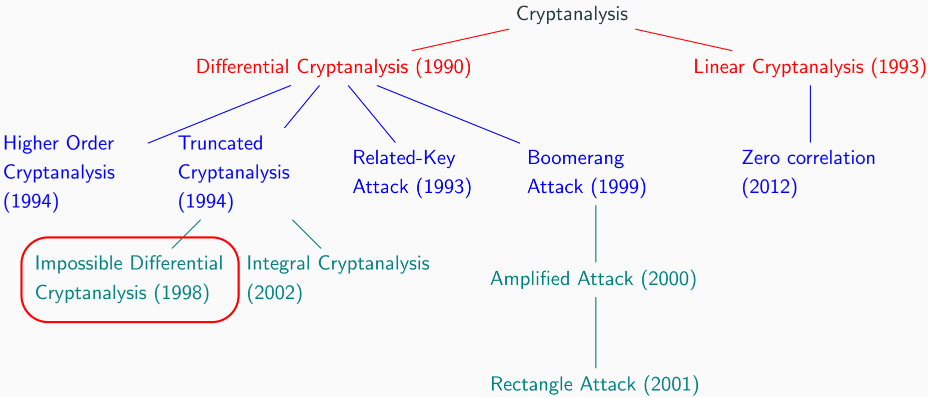
Impossible Differential Cryptanalysis (1998)

Integral Cryptanalysis (2002)

Amplified Attack (2000)

Rectangle Attack (2001)

Variants of Cryptanalysis



- Cryptanalysis
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    - Related-Key Attack (1993)
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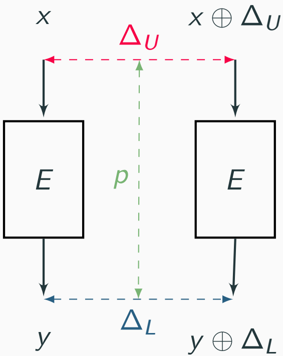
    - Zero correlation (2012)

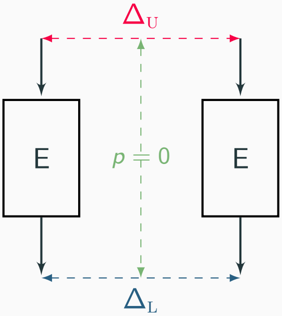
Classical Differential cryptanalysis



- Introduced by [Biham and Shamir, 1990]
- Given an input difference between two plaintexts, some output differences occur more often than others.
- A differential is a pair (Δ_U, Δ_L)

Impossible differential attacks

Exploit differentials of probability 0 (never occur).

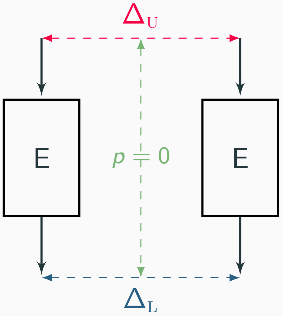


- Find an impossible differential ($\Delta_{\mathrm{U}} \not\to \Delta_{\mathrm{L}}$)

- Build a key-recovery attack

Impossible differential attacks

Exploit differentials of probability 0 (never occur).



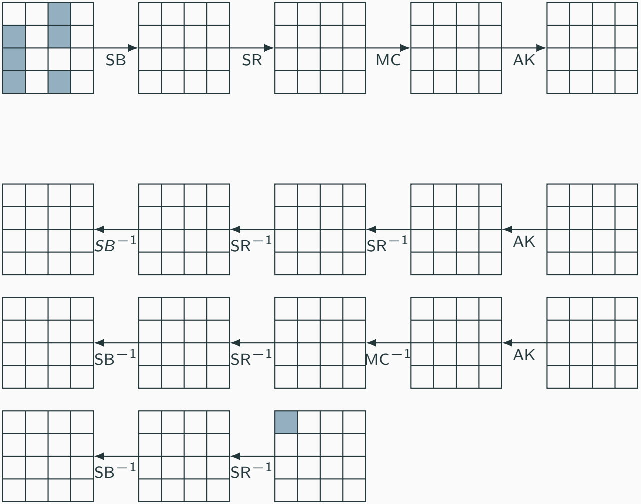
Find an impossible differential ($\Delta_{\mathrm{U}} \not\rightarrow \Delta_{\mathrm{L}}$)

- Miss-in-the-middle technique [Biham et al., 1999]
- U-method [Kim et al., 2003]

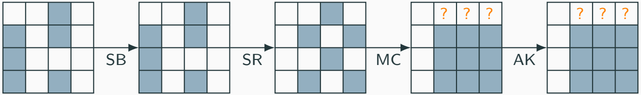
Build a key-recovery attack

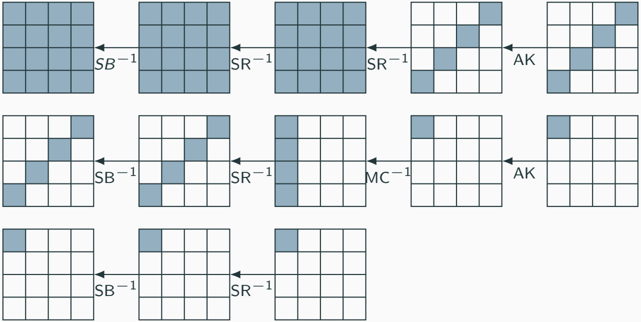
- Early abort technique [Lu et al., 2008]

Miss-in-the-Middle Technique

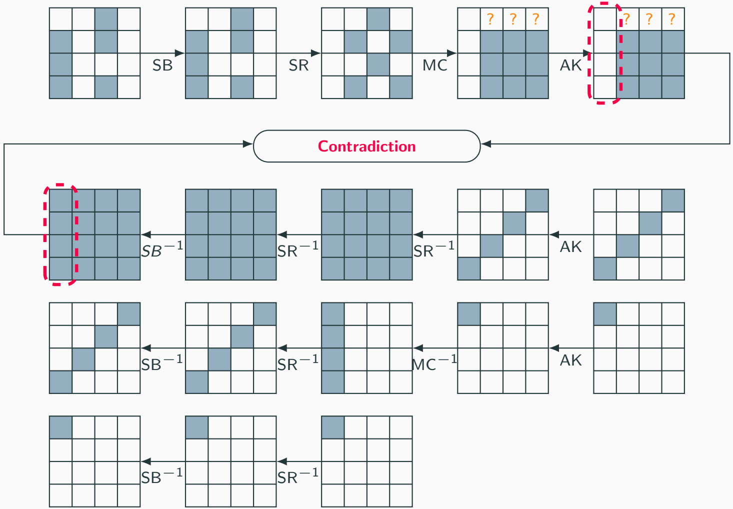


Miss-in-the-Middle Technique



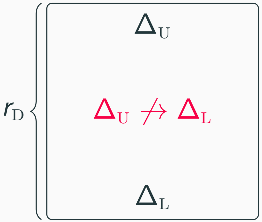


Miss-in-the-Middle Technique



Impossible differential attacks

- 🔍 Find an impossible-differential $\Delta_{\mathrm{U}} \not\rightarrow \Delta_{\mathrm{L}}$
- 🔑 Build a key-recovery attack



r_{D}

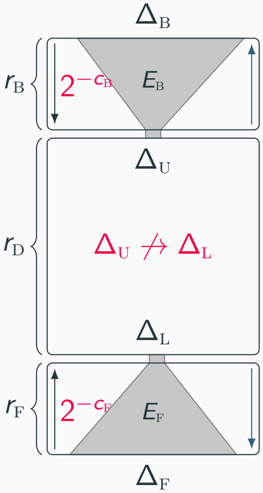
Δ_{U}

$\Delta_{\mathrm{U}} \not\rightarrow \Delta_{\mathrm{L}}$

Δ_{L}

Impossible differential attacks

- 🔍 Find an impossible-differential $\Delta_U \not\rightarrow \Delta_L$
- 🔑 Build a key-recovery attack
  - *Pair Generation.* Generate N pairs satisfying (Δ_B, Δ_F)

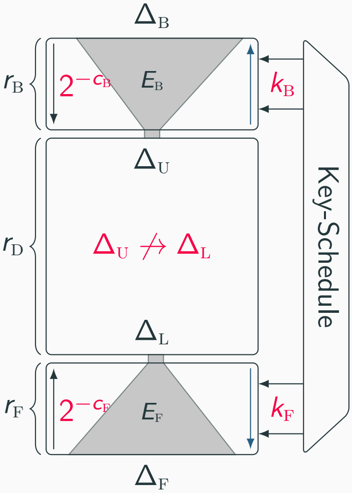


Impossible differential attacks

🔍 Find an impossible-differential $\Delta_{\mathrm{U}} \not\rightarrow \Delta_{\mathrm{L}}$

🔑 Build a key-recovery attack

- *Pair Generation.* Generate N pairs satisfying $(\Delta_{\mathrm{B}}, \Delta_{\mathrm{F}})$
- *Guess-and-Filter.* For all $k \in k_{\mathrm{B}} \cup k_{\mathrm{F}}$:
    - If a pair suggests $(\Delta_{\mathrm{U}}, \Delta_{\mathrm{L}})$, discard k



Impossible differential attacks

- 🔍 Find an impossible-differential $\Delta_{\mathrm{U}} \not\to \Delta_{\mathrm{L}}$
- 🔑 Build a key-recovery attack
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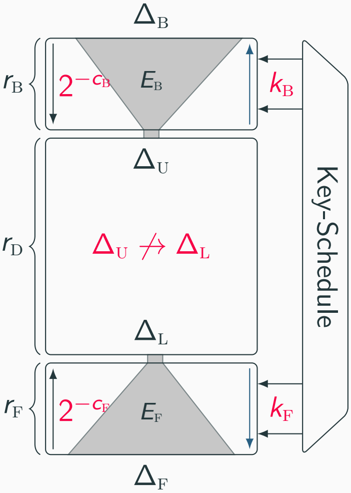
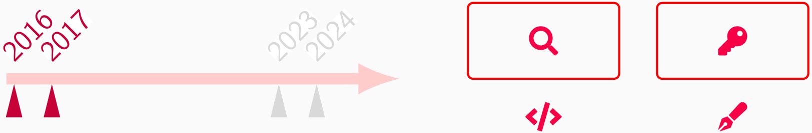
  - *Guess-and-Filter.* For all $k \in k_{\mathrm{B}} \cup k_{\mathrm{F}}$:
    - If a pair suggests $(\Delta_{\mathrm{U}}, \Delta_{\mathrm{L}})$, discard k
  - *Exhaustive Search.* Brute force the remaining key candidates

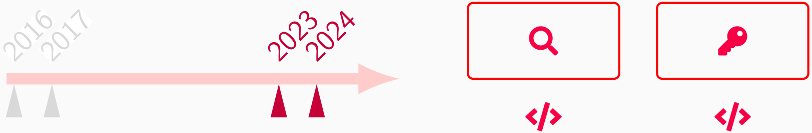


Automatic tools for ID attacks



- Previous works:
    - CRYPTO 2016 [Derbez and Fouque, 2016]
    - Eprint 2016 [Cui et al., 2016]
    - EUROCRYPT 2017 [Sasaki and Todo, 2017]

Automatic tools for ID attacks



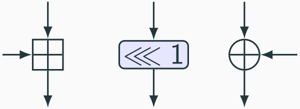
- Previous works:
  - CRYPTO 2016 [Derbez and Fouque, 2016]
  - Eprint 2016 [Cui et al., 2016]
  - EUROCRYPT 2017 [Sasaki and Todo, 2017]
- New approach:
  - EUROCRYPT 2023 [Hadipour et al., 2023]: Introduce the CP approach
  - TOSC 2024 [Hadipour et al., 2024]: Extend to weakly-aligned designs

Motivation

How to **automate** key recovery for complete ID attacks for AndRX and ARX ciphers?

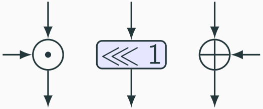
ARX and AndRX designs

ARX



Addition Rotation Xor

AndRX



And Rotation Xor

Our Contributions

- Expanded [Hadipour et al., 2024]'s method
  - Enhanced the model for finding the complicated contradiction
  - Adapted for AndRX and ARX designs
- Proposed a unified model to combine both distinguisher identification (🔍) and key recovery (🔑) for AndRX designs

Overview

Modeling the Distinguishers 🔍

Modeling the Key-Recovery 🔑

Applications

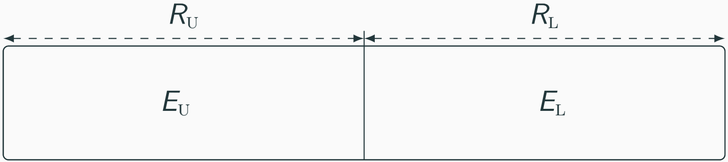
Summary

Modeling the Distinguishers 🔍

Previous Methods to Search for Distinguishers

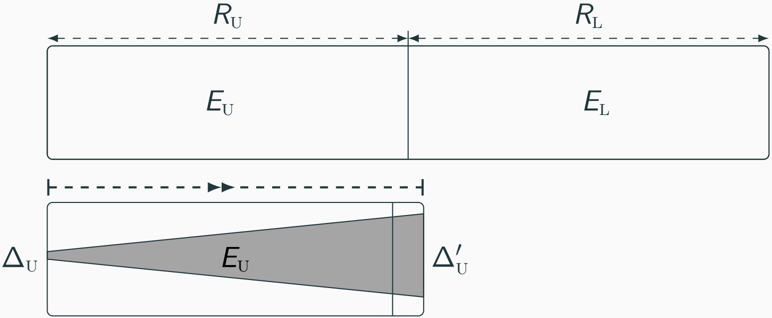
E

Previous Methods to Search for Distinguishers



Previous Methods to Search for Distinguishers

- ✓ $CSP_{\mathrm{U}}(\Delta_{\mathrm{U}}, \Delta'_{\mathrm{U}})$



Previous Methods to Search for Distinguishers

- ✓ $\mathcal{CSP}_{\mathrm{U}}(\Delta_{\mathrm{U}}, \Delta'_{\mathrm{U}})$
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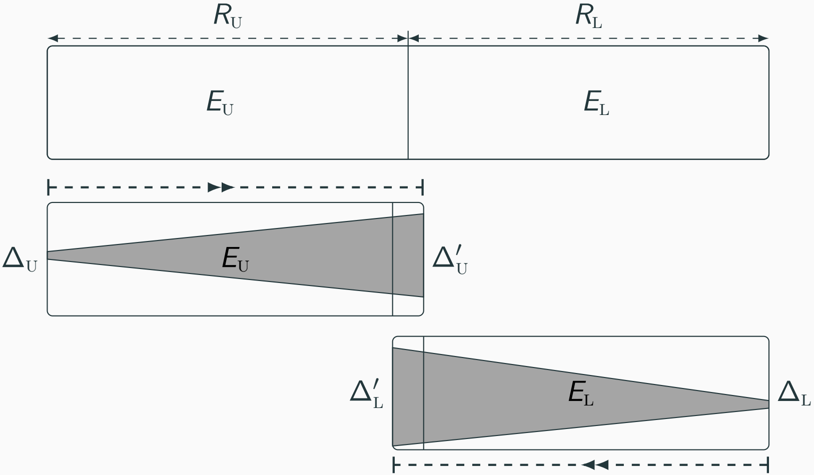
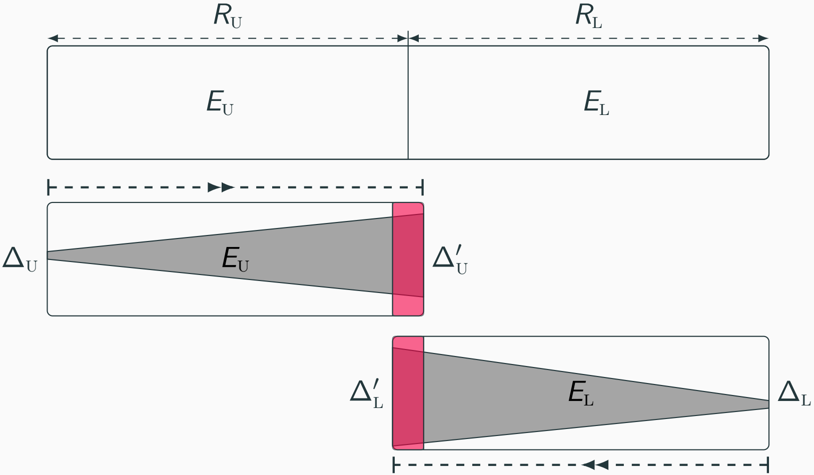


Previous Methods to Search for Distinguishers

- ✓ $CSP_{\mathrm{U}}(\Delta_{\mathrm{U}}, \Delta'_{\mathrm{U}})$
- ✓ $CSP_{\mathrm{L}}(\Delta_{\mathrm{L}}, \Delta'_{\mathrm{L}})$
- ✓ $CSP_{\mathtt{M}}(\Delta'_{\mathrm{U}}, \Delta'_{\mathrm{L}})$



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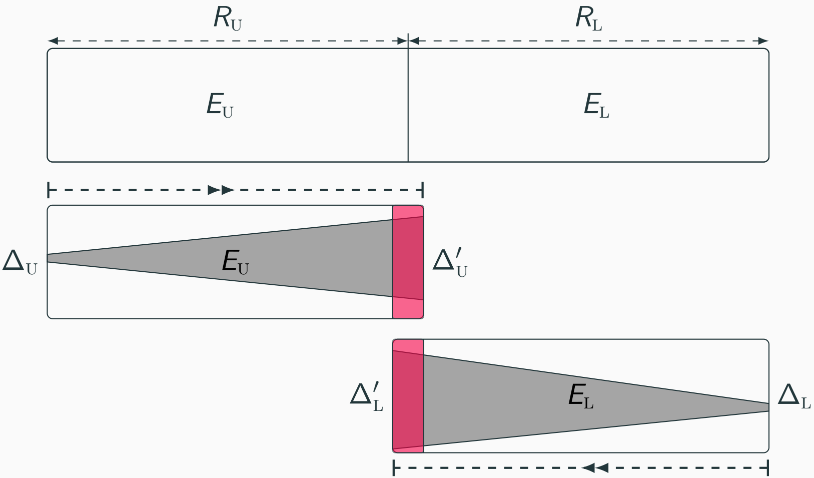

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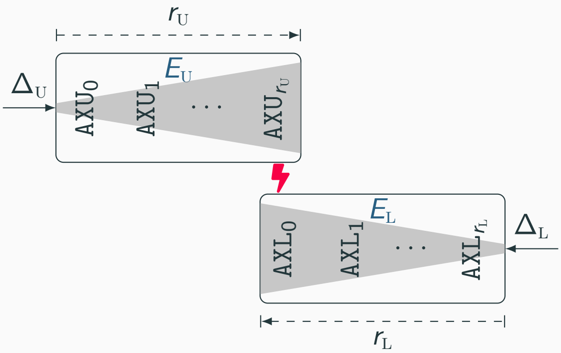
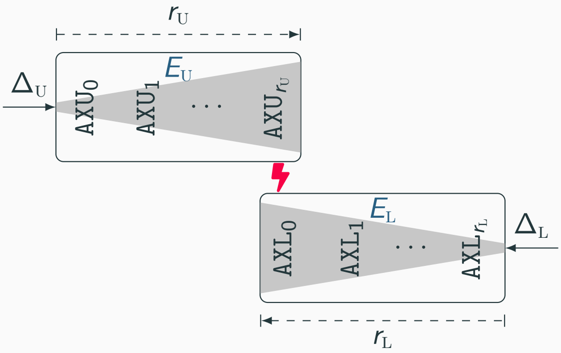


[Hadipour et al., 2023]'s Model.

Previous Methods to Search for Distinguishers

Find ID distinguisher for $r_{\mathrm{D}} (= r_{\mathrm{U}} + r_{\mathrm{L}})$ rounds



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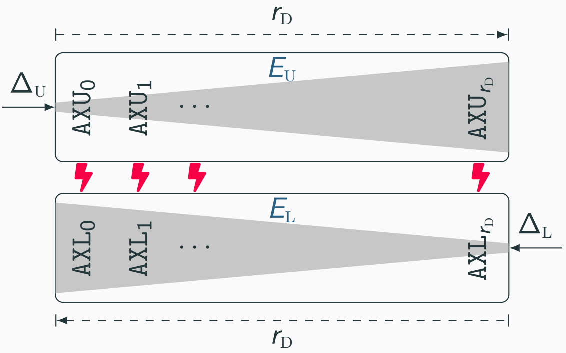


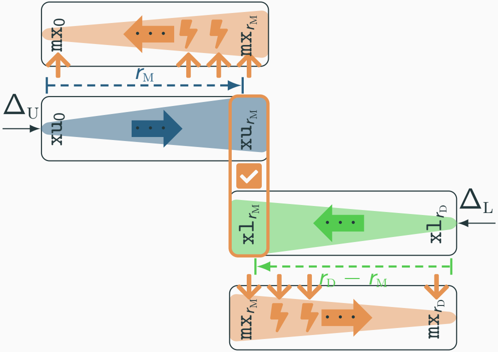
[Hadipour et al., 2024]'s Model

New CP Model to Identify Indirect Contradictions

What if there are no direct contradictions?

- Some contradictions may not be detectable by direct checks.

New CP Model to Identify Indirect Contradictions



- Focus on indirect contradictions described in [Sadeghi and Bagheri, 2018]
- Method Overview:
  - No direct contradiction
  - Merge information from both trails at a specific round
  - Propagate merged information in both directions

CP Model for Deterministic Bit-Wise Trails

- Used to encode the propagation of deterministic differential/linear trails.
- Differences at each bit position encoded via an integer variable.
- Domain: $\{-1, 0, 1\}$.
  - 0: Fixed difference value of 0.
  - 1: Fixed difference value of 1.
  - -1: Difference value is either 0 or 1 (unknown).

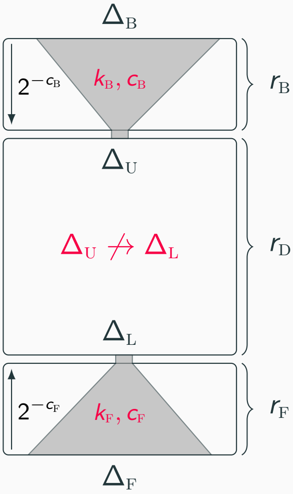
Advanced Bit-wise CP Model for Identifying ID/ZC Distinguishers

- Expand on bit-wise CP model from [Hadipour et al., 2024].
- Introduce new rules for AND and modular addition.
  - And
  - Full adder
  - Modular Addition
- Extends model to detect indirect contradictions beyond direct ones.

Modeling the Key-Recovery 🔑

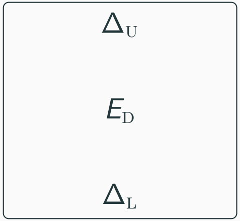
Complexity Analysis of ID Attack [Boura et al., 2018, Boura et al., 2014]

- Number of required pairs: N
- Pair generation: $T_0 = N2^{n+1-|\Delta_{\mathrm{B}}|-|\Delta_{\mathrm{F}}|}$
- Guess-and-filter:
  - $T_1 + T_2 = N + 2^{|k_{\mathrm{B}} \cup k_{\mathrm{F}}|} \frac{N}{2^{c_{\mathrm{B}}+c_{\mathrm{F}}}}$
  - $P = \left(1 - 2^{-(c_{\mathrm{B}}+c_{\mathrm{F}})}\right)^N$
- Exhaustive search: $T_3 = P2^k$
- $T_{tot} = (T_0 + (T_1 + T_2)C_{E'} + T_3)C_E$



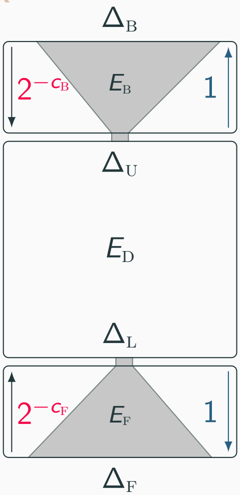
Overall View of Our CP Model for Key-Recovery

- ✓ Model the distinguisher for E_{D} $(\Delta_{\mathrm{U}}, \Delta_{\mathrm{F}})$



Overall View of Our CP Model for Key-Recovery

- ✓ Model the distinguisher for E_{D} $(\Delta_{\mathrm{U}}, \Delta_{\mathrm{F}})$
- ✓ Model the filters in E_{B}, and E_{F} $(c_{\mathrm{B}}, c_{\mathrm{F}}, \Delta_{\mathrm{B}}, \Delta_{\mathrm{F}})$



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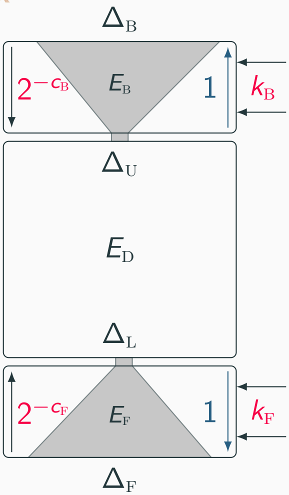
- ✓ Model the guess-and-determine in E_{B}, and E_{F}
- ✓ Model Equivalent Sub-key Technique



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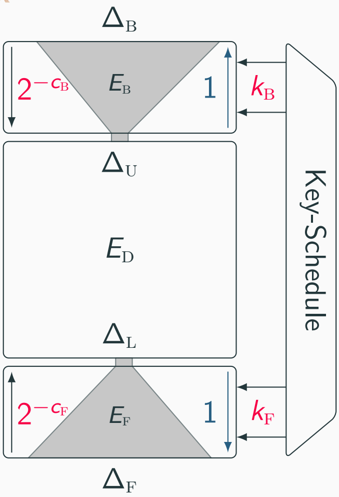
- ✓ Model Equivalent Sub-key Technique
- ✓ Model the complexity formulas



Overall View of Our CP Model for Key-Recovery

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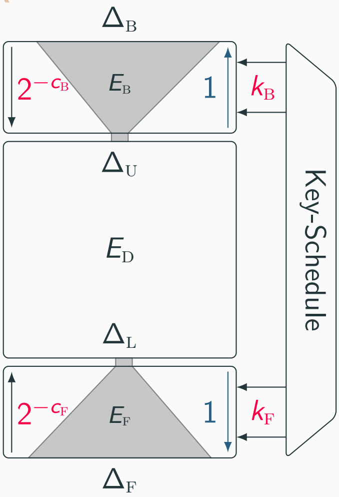
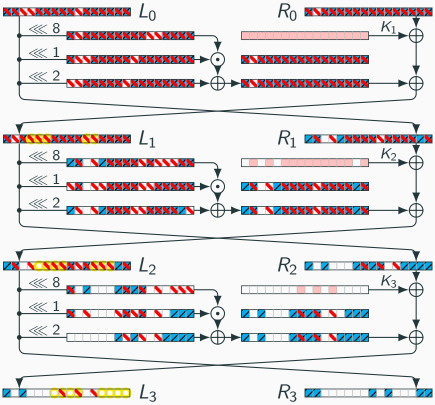
- ✓ Model the guess-and-determine in E_{B}, and E_{F}
- ✓ Model Equivalent Sub-key Technique
- ✓ Model the complexity formulas
- ✓ Objective: `Minimize` the total time complexity

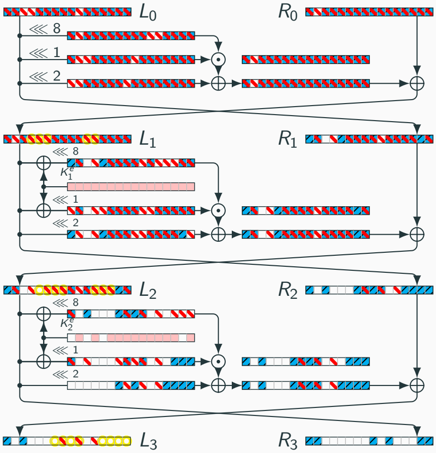


Equivalent Subkey Technique

- Widely used in key-recovery attacks
- Reduces the number of guessed subkey bits
- Methodology:
    - Move K_i of $Round_i$ to $Round_{i+1}$ for $0 \leq i \leq (r_b - 1)$
    - Move K_{i+1} of $Round_i$ to $Round_{i-1}$ for $r_b + r_d \leq i \leq r_b + r_d + r_f - 1$

Equivalent Subkey Technique



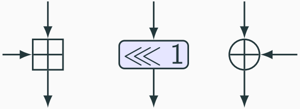


Applications

Applications

ARX

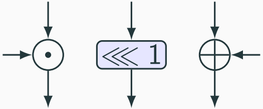
- Block ciphers: LEA, SPECK
- Stream ciphers: ChaCha
- MAC algorithms: SipHash, Chaskey



Addition Rotation Xor

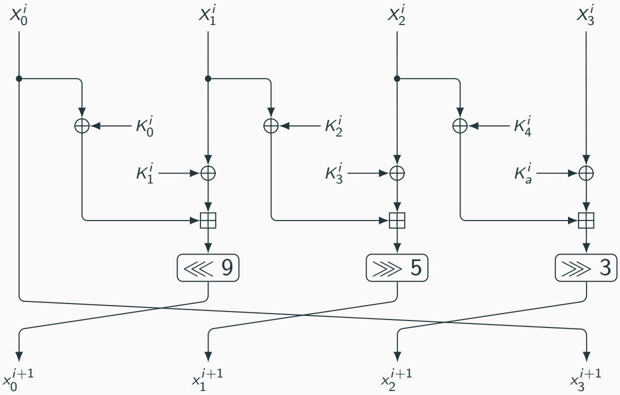
AndRX

- SIMON and Simeck

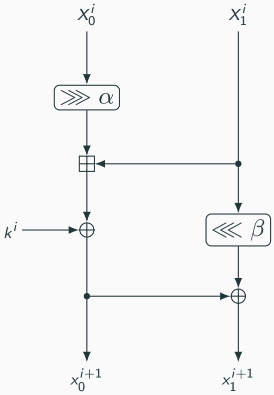


And Rotation Xor

ARX Ciphers

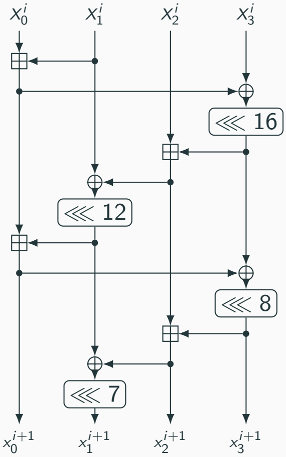


LEA

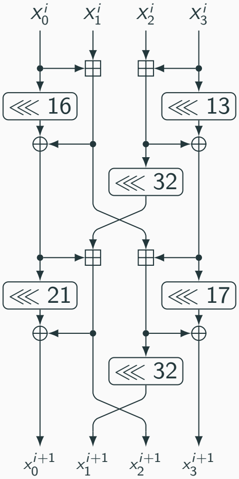


Speck

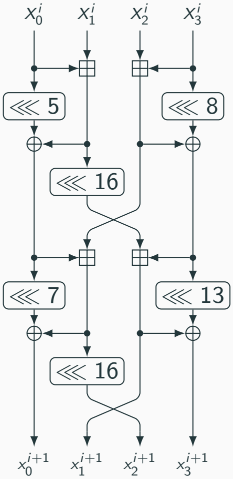
ARX Ciphers



ChaCha



SipHash



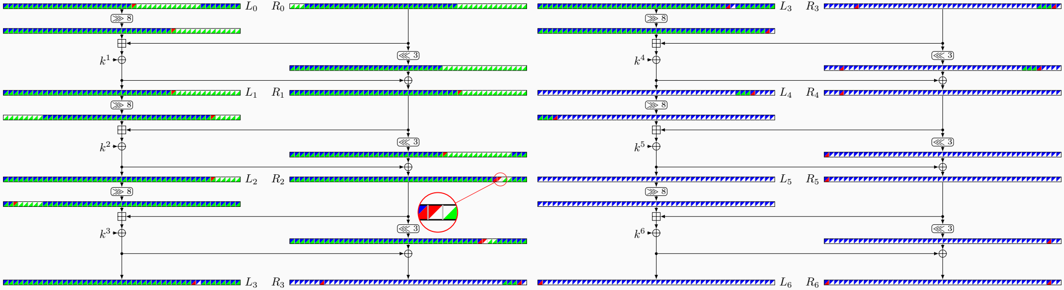
Chaskey

Chaskey

0	00000000000000000000000000000000 *******1000000000000000000000000	00000000000000000000000000000000 00000000000000000000000000000000
1	*******1000000000000000000000000 0000000000000000*******100000000	*******1000000000000000000000000 *******100000000000*******100000
2	******************************** ********************************	*******************************1 ********************************
2	00000000000000000000000000001000 *******************************1	****************************1000 0000000************************1
3	00000000000000000000000000000000 10000000000000000000000000000000	00000000000000000000000000000000 00000000000000000000000000000000
4	10000000000000000000000000000000 00000000000000001000000000000000	10000000000000000000000000000000 10000000000000000001000000000000

Cluster of 2^7 impossible-differential distinguishers for 4-round Chaskey.

Cluster of 2^{65} ID distinguishers for 6-round SPECK-96



🟥 bit difference (linear mask) 1 forward

🟦 unknown difference (linear mask) forward

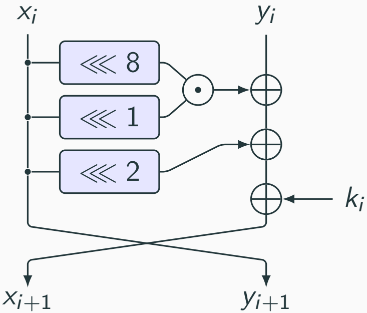
🟥 bit difference (linear mask) 1 backward

🟩 unknown difference (linear mask) backward

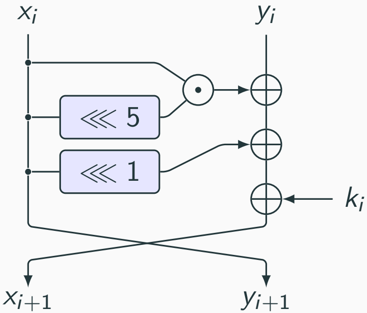
Results

Cipher	Contradiction	#R	#Dist.	Ref.
SPECK-32	Direct	6	3	[Ren and Chen, 2019]
	Direct	6	$\mathbf{2^{4}}$	This work
SPECK-48	Direct	6	20	[Ren and Chen, 2019]
	Direct	6	$\mathbf{2^{17}}$	This work
SPECK-64	Direct	6	157	[Lee et al., 2016, Ren and Chen, 2019]
	Direct	6	$\mathbf{2^{33}}$	This work
SPECK-96	Direct	**6**	$\mathbf{2^{65}}$	This work
SPECK-128	Direct	**6**	$\mathbf{2^{97}}$	This work
LEA	Direct	10	-	[Cui et al., 2016]
	Direct	10	$\mathbf{2^{2}}$	This work
ChaCha	Direct	**5**	$\mathbf{2^{80}}$	This work
SipHash	Direct	**4**	$\mathbf{2^{14}}$	This work
Chaskey	Direct	4	15	[Saberi et al., 2021]
	Direct	4	$\mathbf{2^{7}}$	This work

AndRX Ciphers



SIMON



Simeck

ID Attacks on SIMON

Cipher	#R	Time	Data	Mem.	Ref.
SIMON-32-64	19/20 19/20	$2^{62.56}/2^{62.8}$ $2^{59}/2^{62}$	$2^{32}/2^{32}$ $2^{30.79}/2^{31.47}$	$2^{44}/2^{43.5}$ $2^{47.68}/2^{44.48}$	[Boura et al., 2014, Derbez and Fouque, 2016] This work
SIMON-48-72	20 20	$2^{70.69}$ $2^{67.37}$	2^{48} $2^{46.48}$	2^{58} 2^{64}	[Boura et al., 2014] This work
SIMON-48-96	21 21	$2^{94.73}$ $2^{88.47}$	2^{48} $2^{45.48}$	2^{70} $2^{76.49}$	[Boura et al., 2014] This work
SIMON-64-96	21 21/ 22	$2^{94.56}$ $2^{80.4}/2^{91.81}$	2^{64} $2^{62.79}/2^{63.27}$	2^{60} $2^{71.79}/2^{84.28}$	[Boura et al., 2014] This work
SIMON-64-128	22 22/ 23	$2^{126.56}$ $2^{112.33}/2^{124}$	2^{64} $2^{62.79}/2^{62.47}$	2^{75} $2^{84.78}/2^{99.5}$	[Boura et al., 2014] This work
SIMON-96-96	24 24	$2^{94.62}$ 2^{92}	2^{94} $2^{92.47}$	2^{61} $2^{69.39}$	[Boura et al., 2014] This work
SIMON-96-144	25 25	$2^{142.59}$ $2^{124.793}$	2^{96} $2^{94.793}$	2^{77} $2^{84.785}$	[Boura et al., 2014] This work
SIMON-128-128	27 28	$2^{126.6}$ $2^{114.641}$	2^{94} $2^{110.6}$	2^{61} 2^{86}	[Boura et al., 2014] This work
SIMON-128-192	28 29/30	$2^{190.56}$ $2^{167.42}/2^{181}$	2^{128} $2^{127.278}/2^{127.64}$	2^{77} $2^{97.278}/2^{112.68}$	[Boura et al., 2014] This work
SIMON-128-256	30 30/31	$2^{254.68}$ $2^{235}/2^{251}$	2^{128} $2^{126.86}/2^{124.79}$	2^{111} $2^{112.87}/2^{126.8}$	[Boura et al., 2014] This work

Summary

Conclusions and Future Works

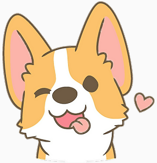
Conclusions

- ☺ Expanded the bit-wise CP model to ARX and AndRX designs
    - Integrated CP model for key recovery in AndRX designs.
- ☺ Introduced a novel model for direct and indirect contradictions.
- ☺ Improved attacks on several ciphers

Future Works

- Apply CP models to other bit-oriented ciphers.
- Enhance optimization techniques for key recovery.

Questions?



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Results on Simeck

Cipher	Dist.	#R	Time	Data	Mem.	Ref.
Simeck-32	11	20	$2^{61.11}$	2^{32}	2^{51}	[Zhang et al., 2007]
	11	20	$\mathbf{2^{55.79}}$	$\mathbf{2^{29.79}}$	$\mathbf{2^{50.79}}$	This work
Simeck-48	$15^{\dagger}$	25	$2^{94.23}$	2^{46}	2^{67}	[Zhang et al., 2007]
	$15^{\dagger}$	25	$\mathbf{2^{93.05}}$	$2^{47.05}$	$2^{68.12}$	This work
Simeck-64	$17^{\dagger}$	27	$2^{126.56}$	2^{63}	2^{68}	[Zhang et al., 2007]
	$17^{\dagger}$	27	$\mathbf{2^{126}}$	$2^{63.47}$	$2^{68.45}$	This work

$\dagger$: Distinguisher based on indirect contradiction.