Safely Doubling your Block Ciphers for a Post-Quantum World

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Outline

- Quantum setting
- Existing quantum attacks
- State of the art of quantum security proofs
- First try and attack
- New construction: QuEME
- Instantiation: Double-AES
- Best attacks on Double-AES

Quantum setting

Quantum algorithms

Grover's search

Grover's search retrieve an element in time $O(\sqrt{n})$ among n other elements.

BHT collision search

BHT algorithm retrieve a collision in time $O(n^{1/3})$ among n elements.

Quantum algorithms

Shor's algorithm

Shor's algorithm solves the factorization problem and discrete logarithm exponentially faster than classical algorithms.

Simon's period finding

Given $f : \{0,1\}^n \rightarrow \{0,1\}^m$ a function which admits a period i.e, there exists $s \in \{0,1\}^n$ such that for all x, y in $\{0,1\}^n$, $f(x) = f(y) \Leftrightarrow x = y$ or $x = y \oplus s$.
Simon's period finding find s in $O(n^3)$ computations.

Quantum models

Classical model

Classical queries and classical computations

$Q1$ model

Classical queries and quantum computations

$Q2$ model

Quantum queries and quantum computations

Quantum memory models

Small quantum memory

Polynomial size or at worst sub-exponential.

QRACM

Classical memory can be queried in superposition.

QRAQM

Quantum memory can be queried in superposition.

Naive impact of quantum computer

Public-key cryptography

The factorization problem and discrete logarithm happen to be the hard problems behind the most used cryptosystems.
$\Rightarrow$ Shor's algorithm breaks them.

Replacement that uses other problems (NIST PQC competition).

Secret-key cryptography

Grover's search speeds up the search for the secret keys.
$\Rightarrow$ Doubling the size of the keys should be enough.

Doubling the state size too [CNS17]

not always enough

$\Rightarrow$ Need for a block cipher with 256-bit key and state (like Saturnin)

LR_5 is the natural proposition but still no proof after many years

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Existing quantum attacks

Even-Mansour cipher

Even-Mansour cipher

Given Π a public n-bit permutation and k_1, k_2 two n-bit secret keys,
$EM : x \mapsto \Pi(x \oplus k_1) \oplus k_2$.
The Even-Mansour cipher is secure against classical attacks up to $2^{n/2}$ computations.

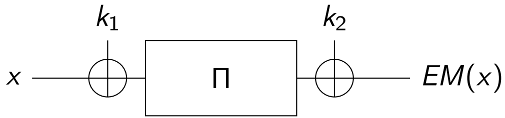
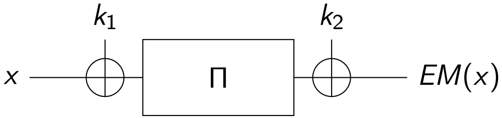


Figure: Even-Mansour Cipher

Attack on Even-Mansour cipher



$Q2$ attack on the Even-Mansour cipher

Observe that $EM(x) \oplus \Pi(x) = \Pi(x \oplus k_1) \oplus k_2 \oplus \Pi(x) = EM(x \oplus k_1) \oplus \Pi(x \oplus k_1)$.
Then Simon's algorithm applied to $x \mapsto EM(x) \oplus \Pi(x)$ retrieves k_1, we can find $k_2 = EM(x) \oplus \Pi(x \oplus k_1)$.

LR_3 (3-round Feistel network)

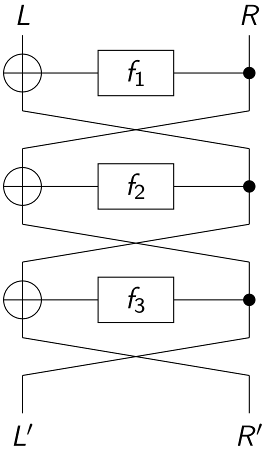
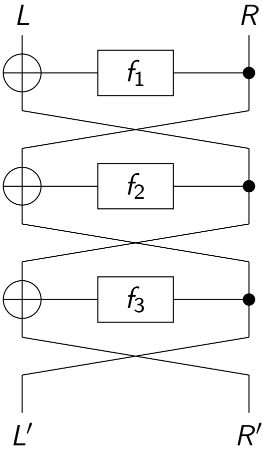


Figure: LR_3 (3-round Feistel network)

Attack on LR_3

$Q2$ attack on LR_3

We fix two values for the right part R_0 and R_1.
Observe that $L'(L, R_\beta) \oplus R_\beta$ only depends on $L \oplus f_1(R_\beta)$.
Then Simon's algorithm applied to $(x, \beta) \mapsto L'(x, R_\beta) \oplus R_\beta$ retrieves $f_1(R_0) \oplus f_1(R_1)$.



Attack on LR_4

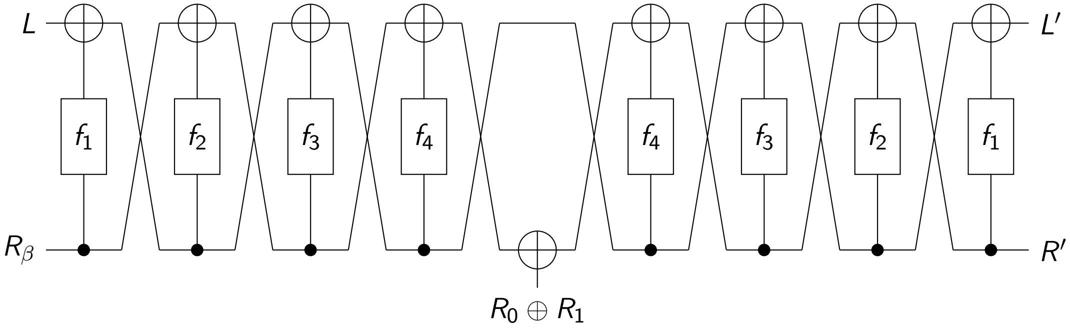


Figure: Attack on LR_4

FX construction

FX-construction

Given E_k a n-bit block cipher and k_1, k_2 two n-bit secret keys,
$EM : x \mapsto E_k(x \oplus k_1) \oplus k_2$.

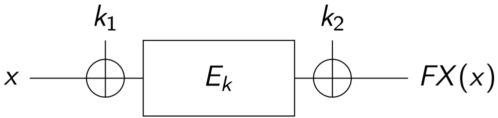
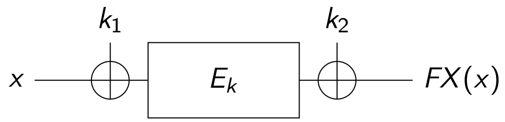


Figure: FX construction

Attack on the FX construction



$Q2$ attack on the FX construction

Like the Even-Mansour cipher, $FX(x) \oplus E_k(x)$ admits k_1 as a period.
Then Simon's algorithm applied to $x \mapsto FX(x) \oplus E_k(x)$ retrieve k_1, we can find $k_2 = FX(x) \oplus E_k(x \oplus k_1)$.

Other quantum attacks

Other Simon-based attacks

- Other constructions have been broken (CBC-MAC, PMAC, GMAC,GCM, OCB,...)
- $Q1$ attacks (Offline-Simon algorithm)
- Linearization attacks

State of the art of quantum security proofs

Polynomial degree minimization

Polynomial degree minimization

For a given quantum algorithm,
build a family of oracles such that:

- The oracles are indexed by few integer variables (one or two in practice).
- The application of the quantum algorithm is a polynomial on the index.
- The family is "large".

Proofs made with this technique

Grover's search and BHT collision search are optimal.

Recording oracle

Recording oracle

For a given attacking quantum algorithm,

- The queries made by the attacker can be recorded as a database (in superposition)
- The superposition can be examined to determine whether it differs from random or not
- We can deduce the advantage of attackers.

Proofs made with this technique

LR_4 is a quantum Pseudo-Random Function.

First Try

First try and attack

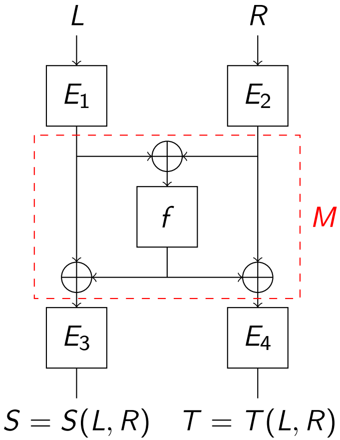


Figure: EME construction

Modification of Simon's algorithm

Simon's period finding

Given $f : \{0,1\}^n \to \{0,1\}^m$ a function which admits a period i.e, there exists $s \in \{0,1\}^n$ such that for all x, y in $\{0,1\}^n$, $f(x) = f(y) \Leftrightarrow x = y$ or $x = y \oplus s$.
Simon's period finding find s in $O(n^3)$ computations.

Quick description of Simon's algorithm

Simon's algorithm starts by making $|\phi_f\rangle = \frac{1}{2^{n/2}} \sum_{x \in \{0,1\}^n} |x\rangle |f(x)\rangle$.
Then it measures the second register and we get a superposition $\frac{1}{\sqrt{2}}(|x_0\rangle + |x_0 + s\rangle)$.
By applying an Hadamard gate, we get a y such that $y \cdot s = 0$.

Modification of Simon's algorithm

Relaxation of Simon's algorithm

It is not necessary to have access to f if we have the superposition $|\phi_f\rangle$.
We can also take a superposition on a restricted space A, $|\phi_{f,A}\rangle = \frac{1}{\sqrt{|A|}} \sum_{x \in A} |x\rangle |f(x)\rangle$.

Practical use

By considering the null function restricted to $A = \{(x, y) | f(x) = f(y)\}$, we can search for s such that $f(x) = f(y) \Rightarrow f(x \oplus s) = f(y \oplus s)$ in $O(n2^{n/3})$ operations.

Attack on EME

$Q2$ Attack

$S(L_0, R_0) = S(L_1, R_1)$ is equivalent to

$$f(E_1(L_0) \oplus E_2(R_0)) \oplus f(E_1(L_1) \oplus E_2(R_1)) = E_1(L_0) \oplus E_1(L_1).$$

If we guess the key of E_2, we can reverse it. Then, by considering the function

$$F : (b, R) \mapsto S(L_b, E_2^{-1}(R)),$$

we can recover $s = (1, E_1(L_0) \oplus E_1(L_1))$ in time $\tilde{O}(2^{k/2} + 2^{n/3})$.

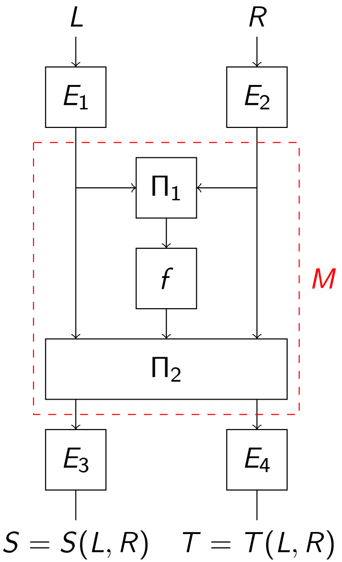
Other weak mixing layer

Other weak mixing layer

Our attack impacts all mixing layers of the form:

$$M(x, y) = \Pi_2(f(\Pi_1(x, y)), x, y)$$

with Π_1 and Π_2 two linear functions. The period is different but the procedure is the same.



New Construction

New construction: QuEME

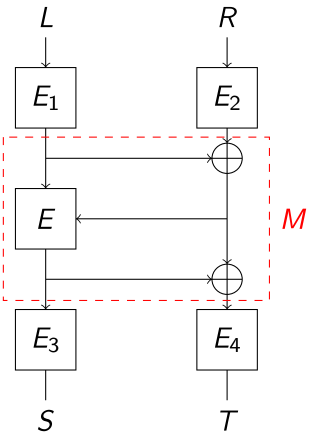


Figure: QuEME construction

Security proofs

Classical security

QuEME is proven to be secure up to 2^n classical queries.

Quantum security

QuEME is proven to be secure up to $2^{n/6}$ quantum queries.

Security claim

We claim QuEME to be secure up to 2^n quantum queries.

Instantiation : Double-AES

Key extension

Key extension

$$K = (k_1 \| k_2)$$
$$k_3 = k_1 \oplus k_2$$
$$k_4 = k_1 \oplus (k_2 \lll 1)$$

Block ciphers

Taking the same block cipher would induce weak keys by having the same permutation for multiple blocks.

Modification of AES-128

Quick description of AES

The state of AES is composed of elements of $\mathbb{F}_{256}$ organized in a 4×4 matrix:

$$\begin{bmatrix} \alpha_0 & \alpha_4 & \alpha_8 & \alpha_{12} \\ \alpha_1 & \alpha_5 & \alpha_9 & \alpha_{13} \\ \alpha_2 & \alpha_6 & \alpha_{10} & \alpha_{14} \\ \alpha_3 & \alpha_7 & \alpha_{11} & \alpha_{15} \end{bmatrix}$$

Modification of AES-128

Composition of a round of AES

AES-128 is composed of 10 rounds which are composed of:

- AddKey xors the state with the round key;
- SubBytes which applies the AES Sbox on all individual elements α_i;
- ShiftRows which shifts the i-th row by i position;
- MixColumns which multiplies each column by a fixed matrix.

The last round omits the Mixcolumns operation and applies one extra AddKey.

Modification of AES-128

AES key schedule

$$K = K_0 = k_0 \| k_1 \| k_2 \| k_3 \text{ and } K_{i+1} = (k_{4i+4} \| k_{4i+5} \| k_{4i+6} \| k_{4i+7}) \text{ for } i \text{ from 0 to 9}$$

$$\begin{aligned}
k_{4i+4} &= \text{SubWord}\left(\text{RotWord}(k_{4i+3})\right) \oplus k_{4i} \oplus rc_i \\
k_{4i+5} &= k_{4i+4} \oplus k_{4i+1} \\
k_{4i+6} &= k_{4i+5} \oplus k_{4i+2} \\
k_{4i+7} &= k_{4i+6} \oplus k_{4i+3}
\end{aligned}$$

$$\text{with } rc_i = \begin{pmatrix} X^i \bmod X^8 + X^4 + X^3 + X + 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Modification of AES-128

We modify the round constant for each block E_j for $j \in \{1,2,3,4\}$:

$$rc_{i,j} = \begin{pmatrix} X^i \bmod X^8 + X^4 + X^3 + X + 1 \\ j \\ 0 \\ 0 \end{pmatrix}$$

Instantiation

Double-AES

We propose Double-AES with 10 rounds of AES for each blocks and claim a unified security claim for both classical and quantum attackers with $T^2/p > 2^{224}$ where T is the time complexity of the attack and p is the probability of success.

Double-AES-7

We conjecture Double-AES-7 (with 7 rounds of AES for each blocks) to also provide the target security.

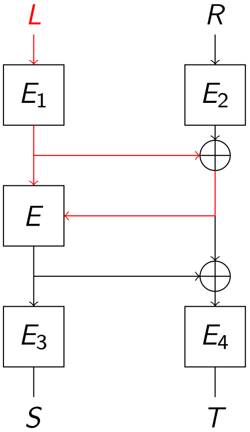
Double-AES-6-MC

We conjecture Double-AES-6-MC (with 6 rounds of AES for each blocks but the last round include a MixColumn operation) to also provide the target security.

Best attacks on Double-AES and preliminary

Cancelation of the first middle round

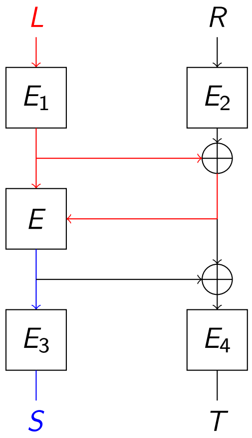
If we introduce a difference in L but not in R, on the first round of the middle encryption, the difference in the plaintext and the first key cancel each other.



Attack on X-2-X

Attack on X-2-X

For a pair of plaintexts $(L_1 \| R)$, $(L_2 \| R)$, we get a ciphertext pair $(S_1 \| T_1)$, $(S_2 \| T_2)$.
From a guess of k_3, we get $E_3^{-1}(S_1) \oplus E_3^{-1}(S_2)$.
From a guess of k_1,we get $E_1(L_1) \oplus E_1(L_2)$ and with 3 bytes of $E_2(R)$, we get 16 possibilities for a byte of $E_3^{-1}(S_1) \oplus E_3^{-1}(S_2)$.



Attack on X-2-X

Using more pairs

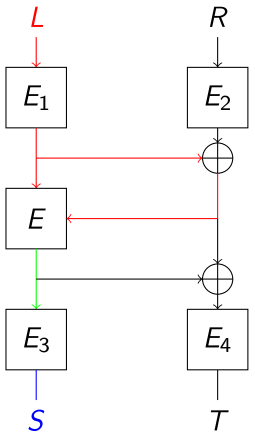
One pair can filter one guess of $(k_1, k_3, E_2(R))$ out of 16. Then by using 70 pairs instead of one, we do not need to guess more elements and filter out every wrong guess.

Complexity

This attack takes $2^{107.5}$ time and memory.

Attack on X-3-3

From a guess of k_1, we can build a set of plaintext (L_i, R) such that $E_1(L_i)$ is constant except the bytes 0 and 8 that take the value i.
We then use the square property of 3-round AES.
We recover the balanced byte from the ciphertext and the guess of 5 bytes of k_3.



Attack on X-3-3

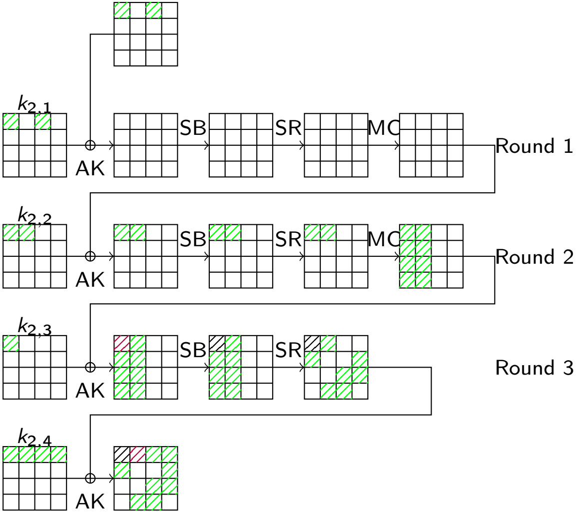


Figure: Square-like property on the middle part.

Attack on X-3-3

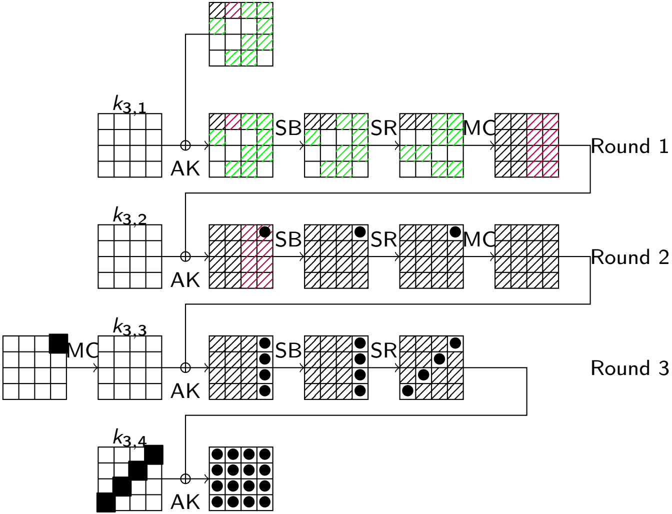


Figure: Recovery on the bottom part.

Attack on X-3-3

Using more sets

One set can filter one guess of out of 256. Then by using 21 sets instead of one, we do not need to guess more elements and filter out every wrong guess.

Complexity

This attack takes $2^{96.5}$ time and data.

Conclusion

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Conclusion

- We extend the properties exploitable for a Simon-based attack and apply it on EME.
- We develop QuEME, a quantum-safe construction for block ciphers.
- We propose Double-AES, a new block-cipher ready for you to experiment.

Future work

Future work

- Other ways to extend the properties retrievable by Simon's algorithm.
- Application of our quantum attack on LR_5
- Further cryptanalysis of Double-AES.