Cluster Search and MILP Modeling for Differential Attacks

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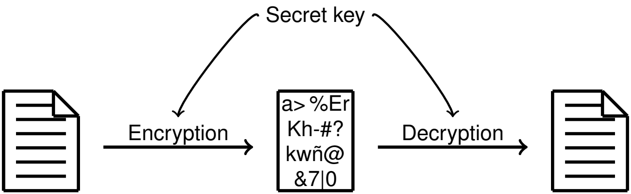
Université de Lorraine, CNRS, Inria, LORIA, Nancy, France

October 27th, 2022

Index

Symmetric Cryptography



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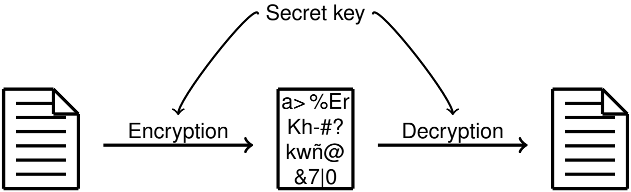

- Stream ciphers
- Block ciphers

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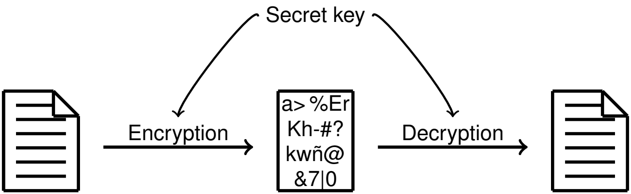

- Block ciphers

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Block Cipher

Given a key $K \in \mathbb{F}_2^m$ and a message $M \in \mathbb{F}_2^N$, a block cipher of block size n is an **invertible** function E_K that encrypts the message M in blocks of size n

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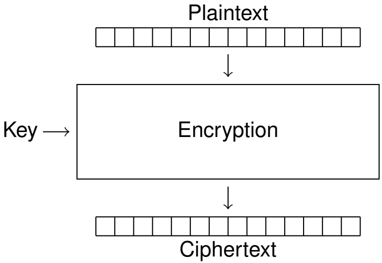

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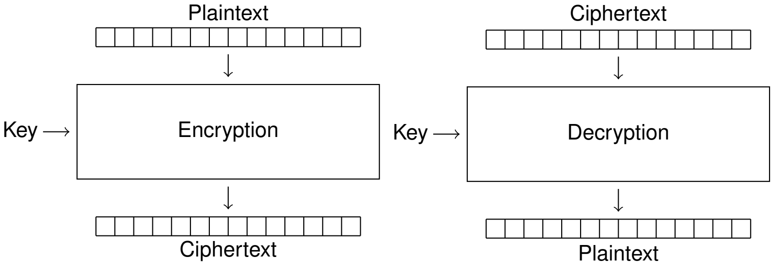

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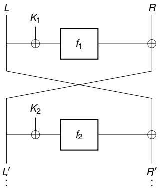

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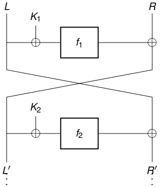

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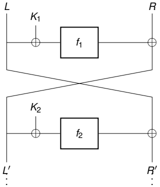

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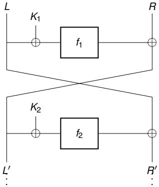

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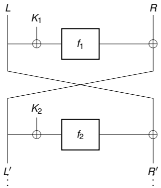

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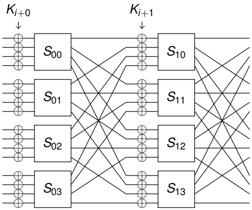

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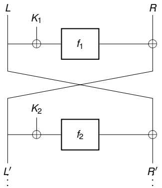

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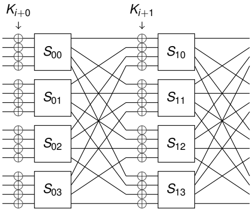

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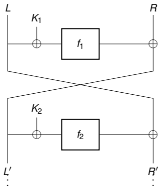

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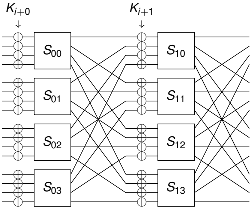

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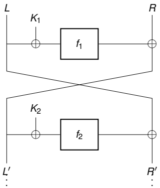

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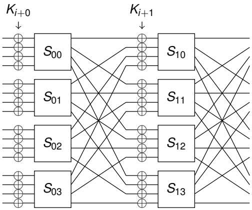

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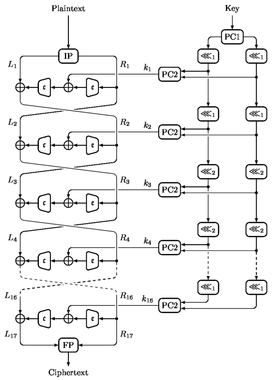
[Jérémy Jean, TikZ for Cryptographers]



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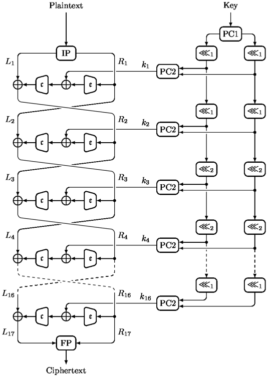

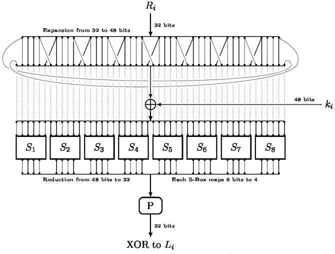

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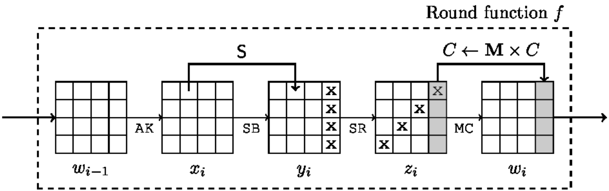

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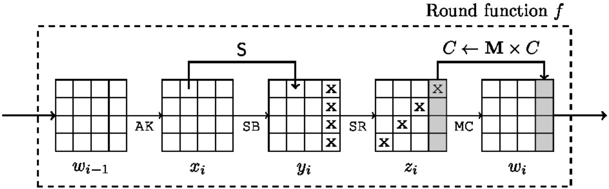

$$\begin{pmatrix} 2 & 3 & 1 & 1 \\ 1 & 2 & 3 & 1 \\ 1 & 1 & 2 & 3 \\ 3 & 1 & 1 & 2 \end{pmatrix}$$

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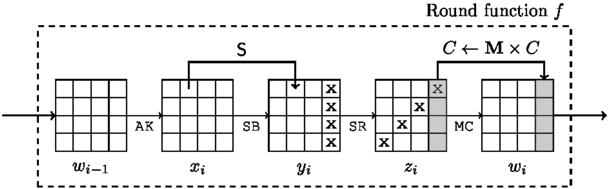

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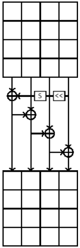

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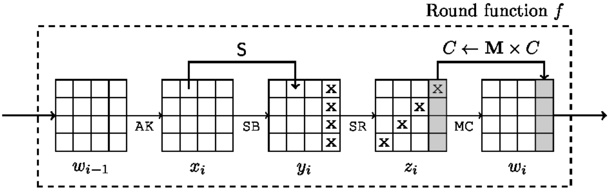

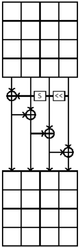

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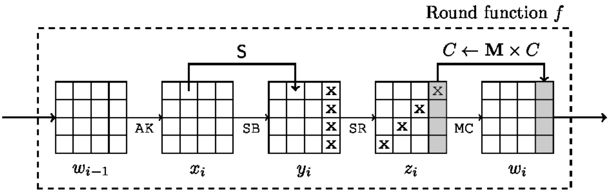

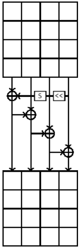

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- Proposed in 1998 by Daemen and Rijmen for the 1997 NIST competition

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Linear operations

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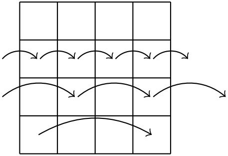

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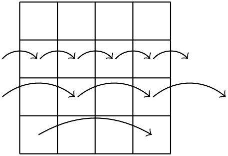

S-box

x	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
S (**x**)	c	a	d	3	e	b	f	7	8	9	1	5	0	2	4	6

Substitution Box

A *substitution box* (*S-box*), is a non-linear operation usually represented as a look-up table:

$$\begin{array}{rccc} \mathbf{S}: & \mathbb{F}_2^{m_1} & \longrightarrow & \mathbb{F}_2^{m_2} \\ & \mathbf{x} & \mapsto & \mathbf{S}(\mathbf{x}) \end{array}$$

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AES S-box:

$\mathbb{F}_{256}$

1. If $x \neq 0$, replace x by its inverse, $x = x^{-1}$ in $\mathbb{F}_2^{8*}$

* Generated by the polynomial $m(X) = X^8 + X^4 + X^3 + X + 1$

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2. $x = Ax + b$, where A is a fix 8×8 binary matrix and b is a fix 8 binary vector

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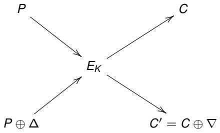
Differential Distinguisher

Find a pair $(\Delta, \nabla) \in \mathbb{F}_2^{2n}$ such that $p(E_K(x \oplus \Delta) = \nabla) \gg 2^{-n}$

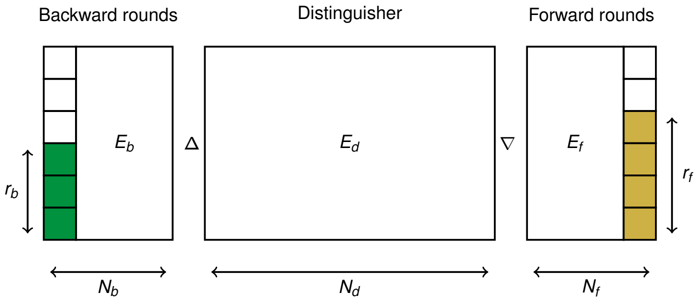
Study the propagation of input differences throughout the cipher:

$$\nabla = E_K(P) \oplus E_K(P \oplus \Delta)$$



Differential Attack

[Biham, E., Shamir, A. Differential cryptanalysis of DES-like cryptosystems. J. Cryptology 4, 3–72 (1991)]



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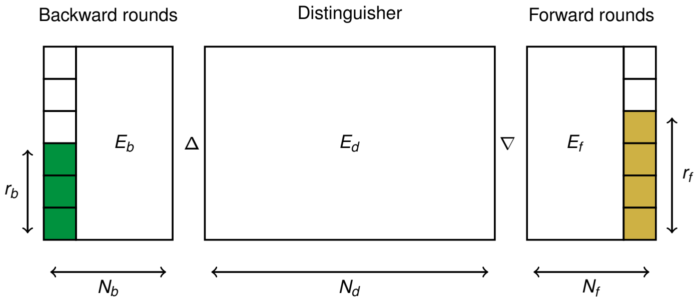

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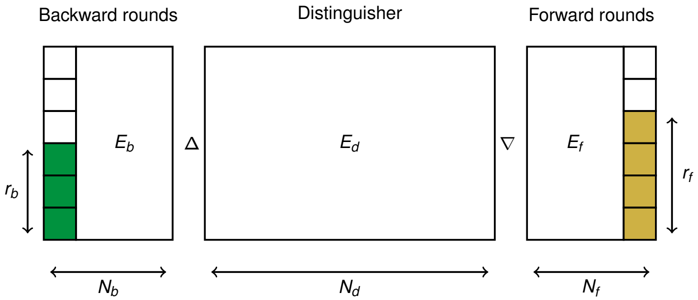

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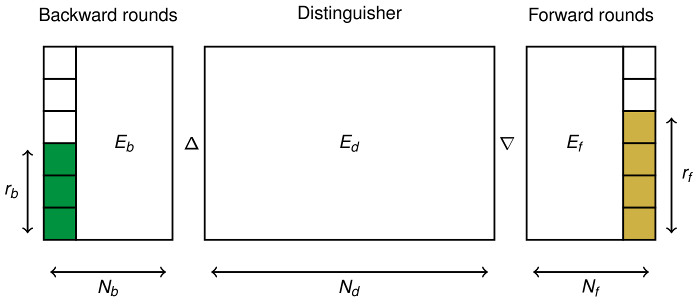

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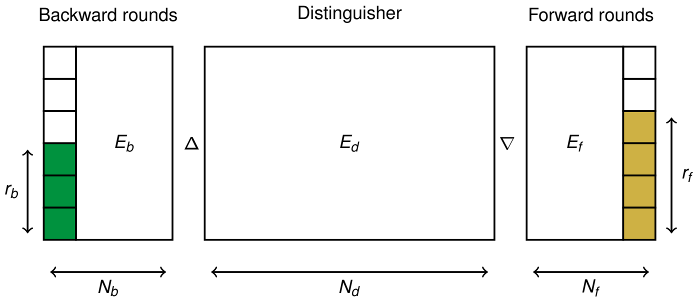

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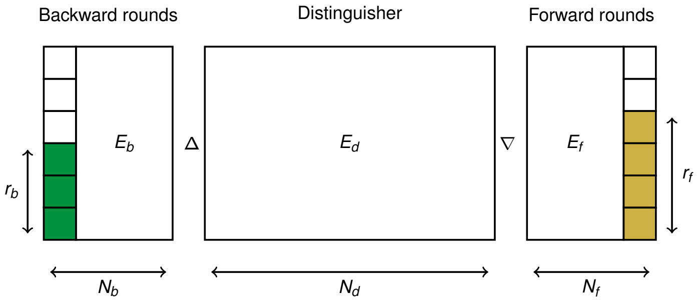

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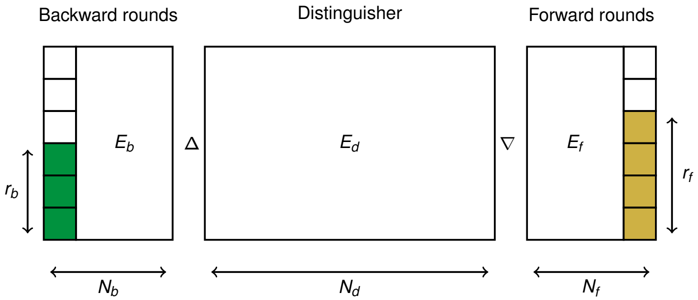

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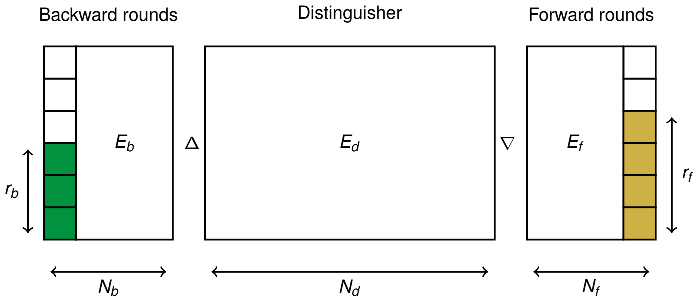

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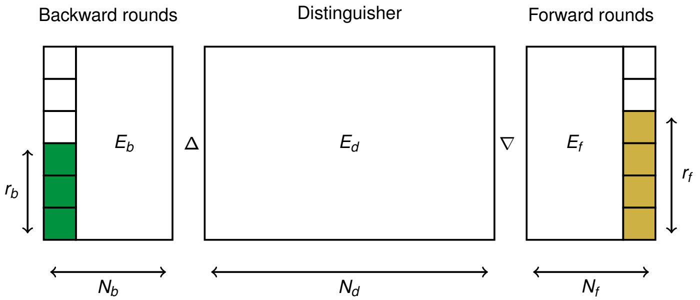

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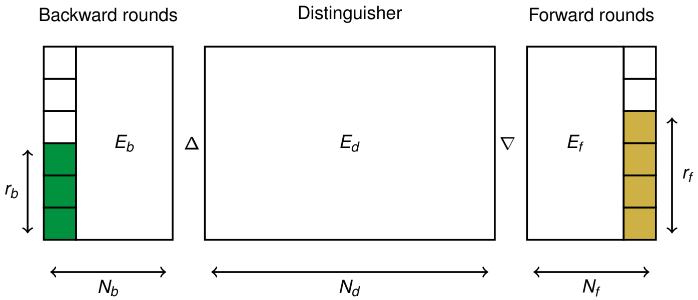
Differential Attack

[Biham, E., Shamir, A. Differential cryptanalysis of DES-like cryptosystems. J. Cryptology 4, 3–72 (1991)]



- Eli Biham and Adi Shamir 1991
- Already known by IBM and some security agencies like NSA
- The pair (Δ, ∇) is referred to as a differential usually hard to find
- Differential characteristic: $\Delta = \delta_0 \rightarrow \delta_1 \rightarrow \cdots \rightarrow \delta_r = \nabla$
- Analyze the differential behavior of cipher operations

Computing the Probability

$$p(\delta_0 \to \delta_1 \cdots \to \delta_r)$$

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$\to$ On the board

Difference Distribution Table

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$$DDT(\Delta_i, \nabla_o) = \#\left\{\mathbf{x} \in \mathbb{F}_2^n : S(\mathbf{x}) \oplus S(\mathbf{x} \oplus \Delta_i) = \nabla_o\right\}$$

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Δ: Input difference	∇: output difference							
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0x0	8	0	0	0	0	0	0	0
0x1	0	2	2	0	0	2	2	0
0x2	0	2	2	0	0	2	2	0
0x3	0	0	0	4	0	0	0	4
0x4	0	0	0	0	4	0	0	4
0x5	0	2	2	0	0	2	2	0
0x6	0	2	2	0	0	2	2	0
0x7	0	0	0	4	4	0	0	0

Abstraction Approach

- Step1: Minimize the number of non-linear *active* operations

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-> Obtain differential characteristics

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A cluster is a set of differential characteristics, for a given number of rounds, that have the same input and output difference

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$$p(\Delta \to \nabla) \approx \sum_j p(\delta_0^j \to \delta_1^j \to \cdots \to \delta_r^j)$$

Implementing Cluster Search

[Chen et all., Improved Differential Characteristic Searching Methods, 2015]

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  - Finds the whole cluster (for small number of rounds)

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- Stores the middle values in a table
- Performs a crossed search
    - Finds the whole cluster (for small number of rounds)
    - Uses too much memory

MILP Modeling

MILP: Mixed-Integer Linear Programming

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- Minimize or maximize an objective function

$$\sum_i a_i X_i$$

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- Constraints $\sum b_i X_i \geq b$ $\sum c_i X_i \leq c$ $\sum d_i X_i == d$

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XOR Truth Table

$a, b, c \in \mathbb{F}_2$

a	b	$c = a \oplus b$
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$(0, 0, 1), (0, 1, 0), (1, 0, 0), (1, 1, 1)$

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$a + b \geq c \qquad a + c \geq b \qquad b + c \geq a$

$a + b + c \leq 2$

Step1 MILP Modeling

Working with truncated characteristics:

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$a + b + c \neq 1 \Rightarrow$

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Step1 objective function:

$$\sum_{i,r} X_{i,r}$$

where i word position, r round, $X_{i,r} = 1$ if there is a non-zero value at the S-box, zero otherwise.

Step2 MILP Modeling

How do we model a non-linear function only with linear constraints?

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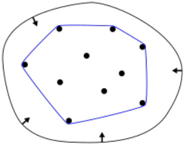
- H-representation of the convex-hull



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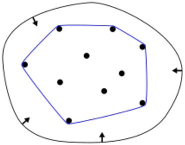

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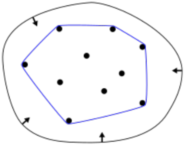

- Product-of-Sum Representation of Boolean Functions

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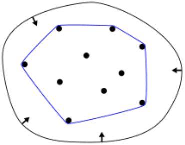

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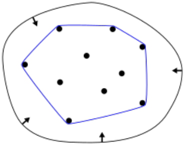

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- Logical condition techniques for 8-bit S-boxes

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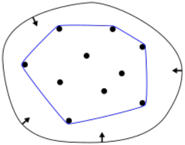

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Minimization

- Greedy algorithm
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- Prime implicants table

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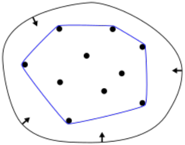

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[Abdelkhalek et al., MILP Modeling for (Large) S-boxes to OptimizeProbability of Differential Characteristics, 2017]

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Δ: Input difference	∇: output difference							
	0x0	0x1	0x2	0x3	0x4	0x5	0x6	0x7
0x0	8	0	0	0	0	0	0	0
0x1	0	2	2	0	0	2	2	0
0x2	0	2	2	0	0	2	2	0
0x3	0	0	0	4	0	0	0	4
0x4	0	0	0	0	4	0	0	4
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- Establish valid transitions: *-DDT

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	0x0	0x1	0x2	0x3	0x4	0x5	0x6	0x7
0x0	1	0	0	0	0	0	0	0
0x1	0	1	1	0	0	1	1	0
0x2	0	1	1	0	0	1	1	0
0x3	0	0	0	1	0	0	0	1
0x4	0	0	0	0	1	0	0	1
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0x2	0	1	1	0	0	1	1	0
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- Use 2-DDT and 4-DDT:

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Δ: Input difference	∇: output difference							
	0x0	0x1	0x2	0x3	0x4	0x5	0x6	0x7
0x0	8	0	0	0	0	0	0	0
0x1	0	0	0	0	0	0	0	0
0x2	0	0	0	0	0	0	0	0
0x3	0	0	0	4	0	0	0	4
0x4	0	0	0	0	4	0	0	4
0x5	0	0	0	0	0	0	0	0
0x6	0	0	0	0	0	0	0	0
0x7	0	0	0	4	4	0	0	0

DDT Modeling with MILP

[Abdelkhalek et al., MILP Modeling for (Large) S-boxes to OptimizeProbability of Differential Characteristics, 2017]

- Establish valid transitions: *-DDT

Δ: Input difference	∇: output difference							
	0x0	0x1	0x2	0x3	0x4	0x5	0x6	0x7
0x0	1	0	0	0	0	0	0	0
0x1	0	1	1	0	0	1	1	0
0x2	0	1	1	0	0	1	1	0
0x3	0	0	0	1	0	0	0	1
0x4	0	0	0	0	1	0	0	1
0x5	0	1	1	0	0	1	1	0
0x6	0	1	1	0	0	1	1	0
0x7	0	0	0	1	1	0	0	0

- Use 2-DDT and 4-DDT:

Δ: Input difference	∇: output difference							
	0x0	0x1	0x2	0x3	0x4	0x5	0x6	0x7
0x0	1	0	0	0	0	0	0	0
0x1	0	1	1	0	0	1	1	0
0x2	0	1	1	0	0	1	1	0
0x3	0	0	0	0	0	0	0	0
0x4	0	0	0	0	0	0	0	0
0x5	0	1	1	0	0	1	1	0
0x6	0	1	1	0	0	1	1	0
0x7	0	0	0	0	0	0	0	0

Δ: Input difference	∇: output difference							
	0x0	0x1	0x2	0x3	0x4	0x5	0x6	0x7
0x0	1	0	0	0	0	0	0	0
0x1	0	0	0	0	0	0	0	0
0x2	0	0	0	0	0	0	0	0
0x3	0	0	0	1	0	0	0	1
0x4	0	0	0	0	1	0	0	1
0x5	0	0	0	0	0	0	0	0
0x6	0	0	0	0	0	0	0	0
0x7	0	0	0	1	1	0	0	0

DDT Modeling with MILP

[Abdelkhalek et al., MILP Modeling for (Large) S-boxes to OptimizeProbability of Differential Characteristics, 2017]

- Establish valid transitions: *-DDT

Δ: Input difference	∇: output difference							
	0x0	0x1	0x2	0x3	0x4	0x5	0x6	0x7
0x0	1	0	0	0	0	0	0	0
0x1	0	1	1	0	0	1	1	0
0x2	0	1	1	0	0	1	1	0
0x3	0	0	0	1	0	0	0	1
0x4	0	0	0	0	1	0	0	1
0x5	0	1	1	0	0	1	1	0
0x6	0	1	1	0	0	1	1	0
0x7	0	0	0	1	1	0	0	0

- Use 2-DDT and 4-DDT:

Δ: Input difference	∇: output difference							
	0x0	0x1	0x2	0x3	0x4	0x5	0x6	0x7
0x0	1	0	0	0	0	0	0	0
0x1	0	1	1	0	0	1	1	0
0x2	0	1	1	0	0	1	1	0
0x3	0	0	0	0	0	0	0	0
0x4	0	0	0	0	0	0	0	0
0x5	0	1	1	0	0	1	1	0
0x6	0	1	1	0	0	1	1	0
0x7	0	0	0	0	0	0	0	0

Δ: Input difference	∇: output difference							
	0x0	0x1	0x2	0x3	0x4	0x5	0x6	0x7
0x0	1	0	0	0	0	0	0	0
0x1	0	0	0	0	0	0	0	0
0x2	0	0	0	0	0	0	0	0
0x3	0	0	0	1	0	0	0	1
0x4	0	0	0	0	1	0	0	1
0x5	0	0	0	0	0	0	0	0
0x6	0	0	0	0	0	0	0	0
0x7	0	0	0	1	1	0	0	0

- H-representation of convex-hull:

DDT Modeling with MILP

[Abdelkhalek et al., MILP Modeling for (Large) S-boxes to OptimizeProbability of Differential Characteristics, 2017]

- Establish valid transitions: *-DDT

Δ: Input difference	∇: output difference							
	0x0	0x1	0x2	0x3	0x4	0x5	0x6	0x7
0x0	1	0	0	0	0	0	0	0
0x1	0	1	1	0	0	1	1	0
0x2	0	1	1	0	0	1	1	0
0x3	0	0	0	1	0	0	0	1
0x4	0	0	0	0	1	0	0	1
0x5	0	1	1	0	0	1	1	0
0x6	0	1	1	0	0	1	1	0
0x7	0	0	0	1	1	0	0	0

- Use 2-DDT and 4-DDT:

Δ: Input difference	∇: output difference							
	0x0	0x1	0x2	0x3	0x4	0x5	0x6	0x7
0x0	1	0	0	0	0	0	0	0
0x1	0	1	1	0	0	1	1	0
0x2	0	1	1	0	0	1	1	0
0x3	0	0	0	0	0	0	0	0
0x4	0	0	0	0	0	0	0	0
0x5	0	1	1	0	0	1	1	0
0x6	0	1	1	0	0	1	1	0
0x7	0	0	0	0	0	0	0	0

Δ: Input difference	∇: output difference							
	0x0	0x1	0x2	0x3	0x4	0x5	0x6	0x7
0x0	1	0	0	0	0	0	0	0
0x1	0	0	0	0	0	0	0	0
0x2	0	0	0	0	0	0	0	0
0x3	0	0	0	1	0	0	0	1
0x4	0	0	0	0	1	0	0	1
0x5	0	0	0	0	0	0	0	0
0x6	0	0	0	0	0	0	0	0
0x7	0	0	0	1	1	0	0	0

- H-representation of convex-hull:
  - Set of valid transitions DDT:

DDT Modeling with MILP

[Abdelkhalek et al., MILP Modeling for (Large) S-boxes to OptimizeProbability of Differential Characteristics, 2017]

- Establish valid transitions: *-DDT

Δ: Input difference	∇: output difference							
	0x0	0x1	0x2	0x3	0x4	0x5	0x6	0x7
0x0	1	0	0	0	0	0	0	0
0x1	0	1	1	0	0	1	1	0
0x2	0	1	1	0	0	1	1	0
0x3	0	0	0	1	0	0	0	1
0x4	0	0	0	0	1	0	0	1
0x5	0	1	1	0	0	1	1	0
0x6	0	1	1	0	0	1	1	0
0x7	0	0	0	1	1	0	0	0

- Use 2-DDT and 4-DDT:

Δ: Input difference	∇: output difference							
	0x0	0x1	0x2	0x3	0x4	0x5	0x6	0x7
0x0	1	0	0	0	0	0	0	0
0x1	0	1	1	0	0	1	1	0
0x2	0	1	1	0	0	1	1	0
0x3	0	0	0	0	0	0	0	0
0x4	0	0	0	0	0	0	0	0
0x5	0	1	1	0	0	1	1	0
0x6	0	1	1	0	0	1	1	0
0x7	0	0	0	0	0	0	0	0

Δ: Input difference	∇: output difference							
	0x0	0x1	0x2	0x3	0x4	0x5	0x6	0x7
0x0	1	0	0	0	0	0	0	0
0x1	0	0	0	0	0	0	0	0
0x2	0	0	0	0	0	0	0	0
0x3	0	0	0	1	0	0	0	1
0x4	0	0	0	0	1	0	0	1
0x5	0	0	0	0	0	0	0	0
0x6	0	0	0	0	0	0	0	0
0x7	0	0	0	1	1	0	0	0

- H-representation of convex-hull:
  - Set of valid transitions DDT:

    $\{(0,0),(1,1),(1,2),(1,5),\ldots,(7,4)\}$

DDT Modeling with MILP

[Abdelkhalek et al., MILP Modeling for (Large) S-boxes to OptimizeProbability of Differential Characteristics, 2017]

- Establish valid transitions: *-DDT

Δ: Input difference	∇: output difference							
	0x0	0x1	0x2	0x3	0x4	0x5	0x6	0x7
0x0	1	0	0	0	0	0	0	0
0x1	0	1	1	0	0	1	1	0
0x2	0	1	1	0	0	1	1	0
0x3	0	0	0	1	0	0	0	1
0x4	0	0	0	0	1	0	0	1
0x5	0	1	1	0	0	1	1	0
0x6	0	1	1	0	0	1	1	0
0x7	0	0	0	1	1	0	0	0

- Use 2-DDT and 4-DDT:

Δ: Input difference	∇: output difference							
	0x0	0x1	0x2	0x3	0x4	0x5	0x6	0x7
0x0	1	0	0	0	0	0	0	0
0x1	0	1	1	0	0	1	1	0
0x2	0	1	1	0	0	1	1	0
0x3	0	0	0	0	0	0	0	0
0x4	0	0	0	0	0	0	0	0
0x5	0	1	1	0	0	1	1	0
0x6	0	1	1	0	0	1	1	0
0x7	0	0	0	0	0	0	0	0

Δ: Input difference	∇: output difference							
	0x0	0x1	0x2	0x3	0x4	0x5	0x6	0x7
0x0	1	0	0	0	0	0	0	0
0x1	0	0	0	0	0	0	0	0
0x2	0	0	0	0	0	0	0	0
0x3	0	0	0	1	0	0	0	1
0x4	0	0	0	0	1	0	0	1
0x5	0	0	0	0	0	0	0	0
0x6	0	0	0	0	0	0	0	0
0x7	0	0	0	1	1	0	0	0

- H-representation of convex-hull:
  - Set of valid transitions DDT:

    $$\{(0,0),(1,1),(1,2),(1,5),\ldots,(7,4)\}$$

  - Set of valid transitions 2-DDT and 4-DDT

DDT Modeling with MILP

[Abdelkhalek et al., MILP Modeling for (Large) S-boxes to OptimizeProbability of Differential Characteristics, 2017]

- Establish valid transitions: *-DDT

Δ: Input difference	∇: output difference							
	0x0	0x1	0x2	0x3	0x4	0x5	0x6	0x7
0x0	1	0	0	0	0	0	0	0
0x1	0	1	1	0	0	1	1	0
0x2	0	1	1	0	0	1	1	0
0x3	0	0	0	1	0	0	0	1
0x4	0	0	0	0	1	0	0	1
0x5	0	1	1	0	0	1	1	0
0x6	0	1	1	0	0	1	1	0
0x7	0	0	0	1	1	0	0	0

- Use 2-DDT and 4-DDT:

Δ: Input difference	∇: output difference							
	0x0	0x1	0x2	0x3	0x4	0x5	0x6	0x7
0x0	1	0	0	0	0	0	0	0
0x1	0	1	1	0	0	1	1	0
0x2	0	1	1	0	0	1	1	0
0x3	0	0	0	0	0	0	0	0
0x4	0	0	0	0	0	0	0	0
0x5	0	1	1	0	0	1	1	0
0x6	0	1	1	0	0	1	1	0
0x7	0	0	0	0	0	0	0	0

Δ: Input difference	∇: output difference							
	0x0	0x1	0x2	0x3	0x4	0x5	0x6	0x7
0x0	1	0	0	0	0	0	0	0
0x1	0	0	0	0	0	0	0	0
0x2	0	0	0	0	0	0	0	0
0x3	0	0	0	1	0	0	0	1
0x4	0	0	0	0	1	0	0	1
0x5	0	0	0	0	0	0	0	0
0x6	0	0	0	0	0	0	0	0
0x7	0	0	0	1	1	0	0	0

- H-representation of convex-hull:
  - Set of valid transitions DDT:

    $$\{(0,0),(1,1),(1,2),(1,5),\ldots,(7,4)\}$$

  - Set of valid transitions 2-DDT and 4-DDT

    $$\{(0,0),(1,1),(1,2),(1,5),\ldots,(6,6)\}$$

DDT Modeling with MILP

[Abdelkhalek et al., MILP Modeling for (Large) S-boxes to OptimizeProbability of Differential Characteristics, 2017]

- Establish valid transitions: *-DDT

Δ: Input difference	∇: output difference							
	0x0	0x1	0x2	0x3	0x4	0x5	0x6	0x7
0x0	1	0	0	0	0	0	0	0
0x1	0	1	1	0	0	1	1	0
0x2	0	1	1	0	0	1	1	0
0x3	0	0	0	1	0	0	0	1
0x4	0	0	0	0	1	0	0	1
0x5	0	1	1	0	0	1	1	0
0x6	0	1	1	0	0	1	1	0
0x7	0	0	0	1	1	0	0	0

- Use 2-DDT and 4-DDT:

Δ: Input difference	∇: output difference							
	0x0	0x1	0x2	0x3	0x4	0x5	0x6	0x7
0x0	1	0	0	0	0	0	0	0
0x1	0	1	1	0	0	1	1	0
0x2	0	1	1	0	0	1	1	0
0x3	0	0	0	0	0	0	0	0
0x4	0	0	0	0	0	0	0	0
0x5	0	1	1	0	0	1	1	0
0x6	0	1	1	0	0	1	1	0
0x7	0	0	0	0	0	0	0	0

Δ: Input difference	∇: output difference							
	0x0	0x1	0x2	0x3	0x4	0x5	0x6	0x7
0x0	1	0	0	0	0	0	0	0
0x1	0	0	0	0	0	0	0	0
0x2	0	0	0	0	0	0	0	0
0x3	0	0	0	1	0	0	0	1
0x4	0	0	0	0	1	0	0	1
0x5	0	0	0	0	0	0	0	0
0x6	0	0	0	0	0	0	0	0
0x7	0	0	0	1	1	0	0	0

- H-representation of convex-hull:
  - Set of valid transitions DDT:

    $$\{(0,0),(1,1),(1,2),(1,5),\ldots,(7,4)\}$$

  - Set of valid transitions 2-DDT and 4-DDT

    $$\{(0,0),(1,1),(1,2),(1,5),\ldots,(6,6)\}$$

    $$\{(0,0),(3,3),(3,7),(4,4),\ldots,(7,4)\}$$

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- Establish valid transitions: *-DDT

Δ: Input difference	∇: output difference							
	0x0	0x1	0x2	0x3	0x4	0x5	0x6	0x7
0x0	1	0	0	0	0	0	0	0
0x1	0	1	1	0	0	1	1	0
0x2	0	1	1	0	0	1	1	0
0x3	0	0	0	1	0	0	0	1
0x4	0	0	0	0	1	0	0	1
0x5	0	1	1	0	0	1	1	0
0x6	0	1	1	0	0	1	1	0
0x7	0	0	0	1	1	0	0	0

- Use 2-DDT and 4-DDT:

Δ: Input difference	∇: output difference							
	0x0	0x1	0x2	0x3	0x4	0x5	0x6	0x7
0x0	1	0	0	0	0	0	0	0
0x1	0	1	1	0	0	1	1	0
0x2	0	1	1	0	0	1	1	0
0x3	0	0	0	0	0	0	0	0
0x4	0	0	0	0	0	0	0	0
0x5	0	1	1	0	0	1	1	0
0x6	0	1	1	0	0	1	1	0
0x7	0	0	0	0	0	0	0	0

Δ: Input difference	∇: output difference							
	0x0	0x1	0x2	0x3	0x4	0x5	0x6	0x7
0x0	1	0	0	0	0	0	0	0
0x1	0	0	0	0	0	0	0	0
0x2	0	0	0	0	0	0	0	0
0x3	0	0	0	1	0	0	0	1
0x4	0	0	0	0	1	0	0	1
0x5	0	0	0	0	0	0	0	0
0x6	0	0	0	0	0	0	0	0
0x7	0	0	0	1	1	0	0	0

- H-representation of convex-hull:
  - Set of valid transitions DDT:

    $$\{(0,0),(1,1),(1,2),(1,5),\ldots,(7,4)\}$$

  - Set of valid transitions 2-DDT and 4-DDT

    $$\{(0,0),(1,1),(1,2),(1,5),\ldots,(6,6)\}$$

    $$\{(0,0),(3,3),(3,7),(4,4),\ldots,(7,4)\}$$

$$\sum_{i=0}^{7} a_i^j x_i + a^j \geq 0$$

MILP Minimization of the H-representation

[Sasaki and Todo, New Algorithm for Modeling S-box in MILP Based Differential and Division Trail Search, 2017]

- Assign a variable to each inequality (Value 0 or 1)

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  - 1 if it is used
  - 0 otherwise
- Relate each inequality to the points that satisfy it
- Minimize the number of inequalities constrained to: all points must be included

- Step2 objective: Maximize the probability of the transitions throughout 2-DDT and 4-DDT

Lightweight Cryptography

Lightweight Cryptography

Need for encryption and authentication on constrained devices

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Need for encryption and authentication on constrained devices

- Small hardware footprint

Lightweight Cryptography

Need for encryption and authentication on constrained devices

- Small hardware footprint
- Small block size

Lightweight Cryptography

Need for encryption and authentication on constrained devices

- Small hardware footprint
- Small block size
- Low-latency

Lightweight Cryptography

Need for encryption and authentication on constrained devices

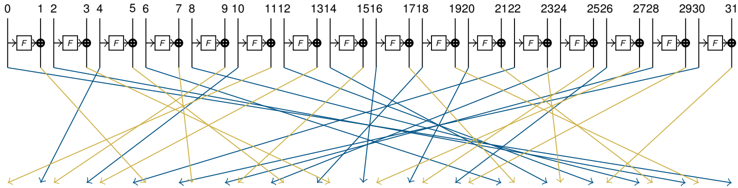
- Small hardware footprint
- Small block size
- Low-latency
- Low-energy consumption

Warp

[Banik et all., WARP : Revisiting GFN for Lightweight 128-bit Block Cipher, 2020]

Warp

[Banik et all., WARP : Revisiting GFN for Lightweight 128-bit Block Cipher, 2020]

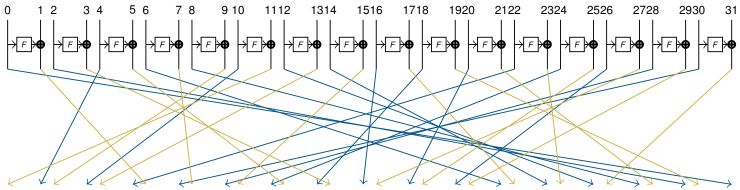


$F = ARK \circ S$

- 128-bit Generalize Feistel cipher

Warp

[Banik et all., WARP : Revisiting GFN for Lightweight 128-bit Block Cipher, 2020]

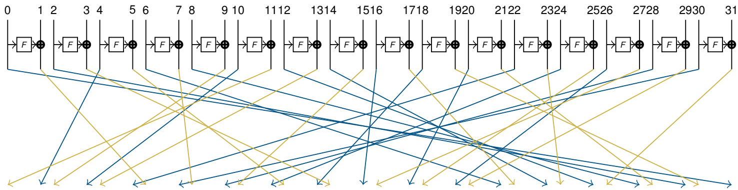


$F = ARK \circ S$

- 128-bit Generalize Feistel cipher
- 128-bit key size

Warp

[Banik et all., WARP : Revisiting GFN for Lightweight 128-bit Block Cipher, 2020]

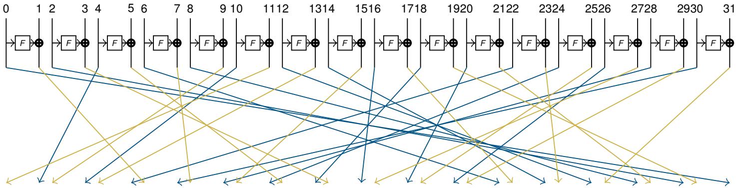


$F = ARK \circ S$

- 128-bit Generalize Feistel cipher
- 128-bit key size
- Linear key schedule

Warp

[Banik et all., WARP : Revisiting GFN for Lightweight 128-bit Block Cipher, 2020]



$F = ARK \circ S$

- 128-bit Generalize Feistel cipher
- 128-bit key size
- Linear key schedule
- 41 round function iterations

Clusters for Warp

Rounds	S-boxes	n_sol	Step2 -log(prob)	Cluster size	Cluster prob
10	17	2	34	4	32
11	22	2	44	4	42
12	28	4	56	16	53
13	34	2	68	512	59

Clusters on Warp

0,	0,	0,	0,	0,	0,	7,	*d*,	*a*,	*d*,	0,	0,	0,	0,	0,	*a*,	0,	0,	0,	0,	0,	0,	0,	0,	*a*,	*d*,	0,	0,	0,	*d*,	0,	0,
0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	*a*,	*a*,	0,	0,	0,	0,	0,	0,	0,	0,	*d*,	7,	0,	0,	0,	0,	0,	*a*,	0,	0,	0,	0,
0,	0,	0,	*a*,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	*a*,	*d*,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,
0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	*a*,	*a*,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,
0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	*a*,	0,	0,	0,	0,	0,	0,	0,	0,
0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	*a*,	0,	0,	0,	0,	0,	0,	0,
0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	*a*,	0,	0,	0,	0,	0,	0,	*a*,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,
a,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	*a*,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	**a**,	0,
0,	0,	0,	0,	*a*,	0,	*a*,	**a**,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	*a*,	0,	0,	0,	0,	*a*,
0,	*a*,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	5,	0,	0,	0,	*a*,	0,	0,	*a*,	0,	*a*,	0,	0,	0,	0,	*a*,	0,	0,	0,	0,	0,
0,	0,	0,	0,	7,	0,	*a*,	0,	0,	0,	0,	0,	0,	0,	0,	*a*,	*a*,	0,	0,	*a*,	0,	0,	*d*,	0,	0,	5,	0,	0,	*a*,	0,	*a*,	0,

Clusters on Warp

0,	0,	0,	0,	0,	0,	7,	*d*,	*a*,	*d*,	0,	0,	0,	0,	0,	*a*,	0,	0,	0,	0,	0,	0,	0,	0,	*a*,	*d*,	0,	0,	0,	*d*,	0,	0,
0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	*a*,	*a*,	0,	0,	0,	0,	0,	0,	0,	0,	*d*,	7,	0,	0,	0,	0,	0,	*a*,	0,	0,	0,	0,
0,	0,	0,	*a*,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	*a*,	*d*,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,
0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	*a*,	*a*,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,
0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	*a*,	0,	0,	0,	0,	0,	0,	0,	0,
0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	*a*,	0,	0,	0,	0,	0,	0,	0,
0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	*a*,	0,	0,	0,	0,	0,	0,	*a*,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,
a,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	*a*,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	**a**,	0,
0,	0,	0,	0,	*a*,	0,	*a*,	**a**,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	*a*,	0,	0,	0,	0,	*a*,
0,	*a*,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	5,	0,	0,	0,	*a*,	0,	0,	*a*,	0,	*a*,	0,	0,	0,	0,	*a*,	0,	0,	0,	0,	0,
0,	0,	0,	0,	7,	0,	*a*,	0,	0,	0,	0,	0,	0,	0,	0,	*a*,	*a*,	0,	0,	*a*,	0,	0,	*d*,	0,	0,	5,	0,	0,	*a*,	0,	*a*,	0,

0,	0,	0,	0,	0,	0,	7,	*d*,	*a*,	*d*,	0,	0,	0,	0,	0,	*a*,	0,	0,	0,	0,	0,	0,	0,	0,	*a*,	*d*,	0,	0,	0,	*d*,	0,	0,
0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	*a*,	*a*,	0,	0,	0,	0,	0,	0,	0,	0,	*d*,	7,	0,	0,	0,	0,	0,	*a*,	0,	0,	0,	0,
0,	0,	0,	*a*,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	*a*,	*d*,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,
0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	*a*,	*a*,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,
0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	*a*,	0,	0,	0,	0,	0,	0,	0,	0,
0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	*a*,	0,	0,	0,	0,	0,	0,	0,
0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	*a*,	0,	0,	0,	0,	0,	0,	*a*,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,
a,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	*a*,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	**5**,	0,
0,	0,	0,	0,	*a*,	0,	*a*,	**5**,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	*a*,	0,	0,	0,	0,	*a*,
0,	*a*,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	5,	0,	0,	0,	*a*,	0,	0,	*a*,	0,	*a*,	0,	0,	0,	0,	*a*,	0,	0,	0,	0,	0,
0,	0,	0,	0,	7,	0,	*a*,	0,	0,	0,	0,	0,	0,	0,	0,	*a*,	*a*,	0,	0,	*a*,	0,	0,	*d*,	0,	0,	5,	0,	0,	*a*,	0,	*a*,	0,

Clusters on Warp

0,	0,	0,	0,	0,	0,	7,	*d*,	*a*,	*d*,	0,	0,	0,	0,	0,	*a*,	0,	0,	0,	0,	0,	0,	0,	0,	*a*,	*d*,	0,	0,	0,	*d*,	0,	0,
0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	*a*,	*a*,	0,	0,	0,	0,	0,	0,	0,	0,	*d*,	7,	0,	0,	0,	0,	0,	*a*,	0,	0,	0,	0,
0,	0,	0,	*a*,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	*a*,	*d*,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,
0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	*a*,	*a*,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,
0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	*a*,	0,	0,	0,	0,	0,	0,	0,	0,
0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	*a*,	0,	0,	0,	0,	0,	0,	0,
0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	*a*,	0,	0,	0,	0,	0,	0,	*a*,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,
a,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	*a*,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	**a**,	0,
0,	0,	0,	0,	*a*,	0,	*a*,	**a**,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	*a*,	0,	0,	0,	0,	*a*,
0,	*a*,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	5,	0,	0,	0,	*a*,	0,	0,	*a*,	0,	*a*,	0,	0,	0,	0,	*a*,	0,	0,	0,	0,	0,
0,	0,	0,	0,	7,	0,	*a*,	0,	0,	0,	0,	0,	0,	0,	0,	*a*,	*a*,	0,	0,	*a*,	0,	0,	*d*,	0,	0,	5,	0,	0,	*a*,	0,	*a*,	0,

0,	0,	0,	0,	0,	0,	7,	*d*,	*a*,	*d*,	0,	0,	0,	0,	0,	*a*,	0,	0,	0,	0,	0,	0,	0,	0,	*a*,	*d*,	0,	0,	0,	*d*,	0,	0,
0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	*a*,	*a*,	0,	0,	0,	0,	0,	0,	0,	0,	*d*,	7,	0,	0,	0,	0,	0,	*a*,	0,	0,	0,	0,
0,	0,	0,	*a*,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	*a*,	*d*,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,
0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	*a*,	*a*,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,
0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	*a*,	0,	0,	0,	0,	0,	0,	0,	0,
0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	*a*,	0,	0,	0,	0,	0,	0,	0,
0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	*a*,	0,	0,	0,	0,	0,	0,	*a*,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,
a,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	*a*,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	**5**,	0,
0,	0,	0,	0,	*a*,	0,	*a*,	**5**,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	*a*,	0,	0,	0,	0,	*a*,
0,	*a*,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	5,	0,	0,	0,	*a*,	0,	0,	*a*,	0,	*a*,	0,	0,	0,	0,	*a*,	0,	0,	0,	0,	0,
0,	0,	0,	0,	7,	0,	*a*,	0,	0,	0,	0,	0,	0,	0,	0,	*a*,	*a*,	0,	0,	*a*,	0,	0,	*d*,	0,	0,	5,	0,	0,	*a*,	0,	*a*,	0,

0,	0,	0,	0,	0,	0,	7,	*d*,	*a*,	*d*,	0,	0,	0,	0,	0,	*a*,	0,	0,	0,	0,	0,	0,	0,	0,	*a*,	*d*,	0,	0,	0,	*d*,	0,	0,
0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	*a*,	*a*,	0,	0,	0,	0,	0,	0,	0,	0,	*d*,	7,	0,	0,	0,	0,	0,	*a*,	0,	0,	0,	0,
0,	0,	0,	*a*,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	*a*,	*d*,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,
0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	*a*,	*a*,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,
0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	*a*,	0,	0,	0,	0,	0,	0,	0,	0,
0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	*a*,	0,	0,	0,	0,	0,	0,	0,
0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	*a*,	0,	0,	0,	0,	0,	0,	*a*,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,
a,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	*a*,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	**d**,	0,
0,	0,	0,	0,	*a*,	0,	*a*,	**d**,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	*a*,	0,	0,	0,	0,	*a*,
0,	*a*,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	5,	0,	0,	0,	*a*,	0,	0,	*a*,	0,	*a*,	0,	0,	0,	0,	*a*,	0,	0,	0,	0,	0,
0,	0,	0,	0,	7,	0,	*a*,	0,	0,	0,	0,	0,	0,	0,	0,	*a*,	*a*,	0,	0,	*a*,	0,	0,	*d*,	0,	0,	5,	0,	0,	*a*,	0,	*a*,	0,

Clusters on Warp

0,	0,	0,	0,	0,	0,	7,	d,	a,	d,	0,	0,	0,	0,	0,	a,	0,	0,	0,	0,	0,	0,	0,	0,	a,	d,	0,	0,	0,	d,	0,	0,
0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	a,	a,	0,	0,	0,	0,	0,	0,	0,	0,	d,	7,	0,	0,	0,	0,	0,	a,	0,	0,	0,	0,
0,	0,	0,	a,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	a,	d,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,
0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	a,	a,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,
0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	a,	0,	0,	0,	0,	0,	0,	0,	0,
0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	a,	0,	0,	0,	0,	0,	0,	0,
0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	a,	0,	0,	0,	0,	0,	0,	a,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,
a,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	a,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	**a**,	0,
0,	0,	0,	0,	a,	0,	a,	**a**,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	a,	0,	0,	0,	0,	a,
0,	a,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	5,	0,	0,	0,	a,	0,	0,	a,	0,	a,	0,	0,	0,	0,	a,	0,	0,	0,	0,	0,
0,	0,	0,	0,	7,	0,	a,	0,	0,	0,	0,	0,	0,	0,	0,	a,	a,	0,	0,	a,	0,	0,	d,	0,	0,	5,	0,	0,	a,	0,	a,	0,

0,	0,	0,	0,	0,	0,	7,	d,	a,	d,	0,	0,	0,	0,	0,	a,	0,	0,	0,	0,	0,	0,	0,	0,	a,	d,	0,	0,	0,	d,	0,	0,
0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	a,	a,	0,	0,	0,	0,	0,	0,	0,	0,	d,	7,	0,	0,	0,	0,	0,	a,	0,	0,	0,	0,
0,	0,	0,	a,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	a,	d,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,
0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	a,	a,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,
0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	a,	0,	0,	0,	0,	0,	0,	0,	0,
0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	a,	0,	0,	0,	0,	0,	0,	0,
0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	a,	0,	0,	0,	0,	0,	0,	a,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,
a,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	a,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	**5**,	0,
0,	0,	0,	0,	a,	0,	a,	**5**,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	a,	0,	0,	0,	0,	a,
0,	a,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	5,	0,	0,	0,	a,	0,	0,	a,	0,	a,	0,	0,	0,	0,	a,	0,	0,	0,	0,	0,
0,	0,	0,	0,	7,	0,	a,	0,	0,	0,	0,	0,	0,	0,	0,	a,	a,	0,	0,	a,	0,	0,	d,	0,	0,	5,	0,	0,	a,	0,	a,	0,

0,	0,	0,	0,	0,	0,	7,	d,	a,	d,	0,	0,	0,	0,	0,	a,	0,	0,	0,	0,	0,	0,	0,	0,	a,	d,	0,	0,	0,	d,	0,	0,
0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	a,	a,	0,	0,	0,	0,	0,	0,	0,	0,	d,	7,	0,	0,	0,	0,	0,	a,	0,	0,	0,	0,
0,	0,	0,	a,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	a,	d,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,
0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	a,	a,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,
0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	a,	0,	0,	0,	0,	0,	0,	0,	0,
0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	a,	0,	0,	0,	0,	0,	0,	0,
0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	a,	0,	0,	0,	0,	0,	0,	a,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,
a,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	a,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	**d**,	0,
0,	0,	0,	0,	a,	0,	a,	**d**,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	a,	0,	0,	0,	0,	a,
0,	a,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	5,	0,	0,	0,	a,	0,	0,	a,	0,	a,	0,	0,	0,	0,	a,	0,	0,	0,	0,	0,
0,	0,	0,	0,	7,	0,	a,	0,	0,	0,	0,	0,	0,	0,	0,	a,	a,	0,	0,	a,	0,	0,	d,	0,	0,	5,	0,	0,	a,	0,	a,	0,

0,	0,	0,	0,	0,	0,	7,	d,	a,	d,	0,	0,	0,	0,	0,	a,	0,	0,	0,	0,	0,	0,	0,	0,	a,	d,	0,	0,	0,	d,	0,	0,
0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	a,	a,	0,	0,	0,	0,	0,	0,	0,	0,	d,	7,	0,	0,	0,	0,	0,	a,	0,	0,	0,	0,
0,	0,	0,	a,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	a,	d,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,
0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	a,	a,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,
0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	a,	0,	0,	0,	0,	0,	0,	0,	0,
0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	a,	0,	0,	0,	0,	0,	0,	0,
0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	a,	0,	0,	0,	0,	0,	0,	a,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,
a,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	a,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	**f**,	0,
0,	0,	0,	0,	a,	0,	a,	**f**,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	a,	0,	0,	0,	0,	a,
0,	a,	0,	0,	0,	0,	0,	0,	0,	0,	0,	0,	5,	0,	0,	0,	a,	0,	0,	a,	0,	a,	0,	0,	0,	0,	a,	0,	0,	0,	0,	0,
0,	0,	0,	0,	7,	0,	a,	0,	0,	0,	0,	0,	0,	0,	0,	a,	a,	0,	0,	a,	0,	0,	d,	0,	0,	5,	0,	0,	a,	0,	a,	0,

Skinny

[Beierle et all., The SKINNY Family of Block Ciphers and its Low-Latency Variant MANTIS, 2016]

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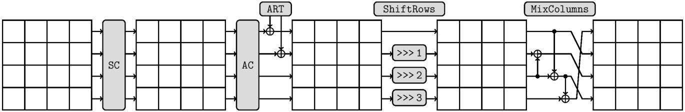
Round function:



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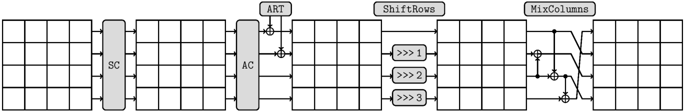

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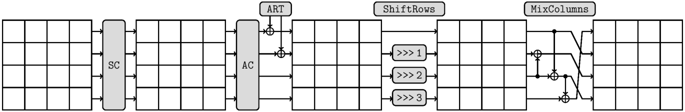

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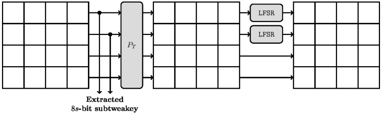

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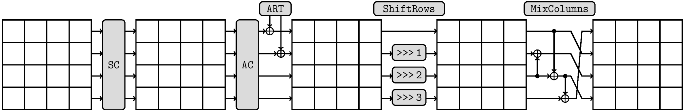

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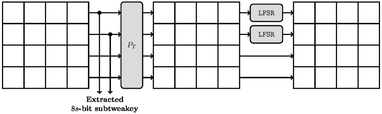

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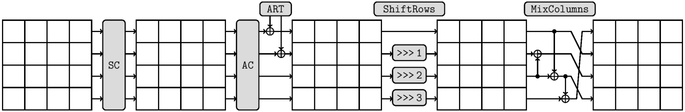

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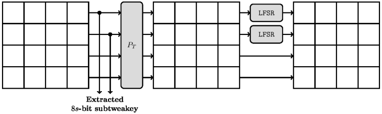

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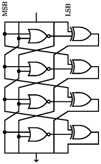

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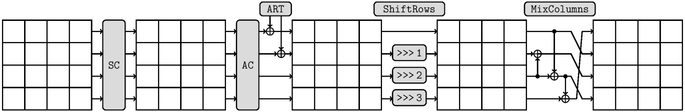

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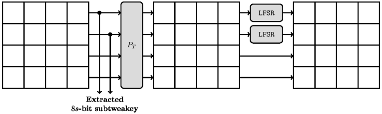

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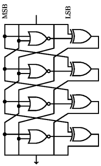

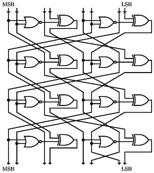

Step1 Skinny

[Delaune et all., SKINNY with Scalpel - Comparing Tools for Differential Analysis, 2020]

Rounds	SK	n_sol	TK1	n_sol	TK2	n_sol	TK3	n_sol
11	52	2	32	2	16	1	10	3
12	55	2	38	7	21	1	13	2
13	58	6	41	2	25	2	16	2
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- Proposed solution: Add extra variables to control instantiation probability

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- Need fo better Step1 model for Skinny

Cluster Search and MILP Modeling for Differential Attacks

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Université de Lorraine, CNRS, Inria, LORIA, Nancy, France

October 27th, 2022